

期末試験・水曜・午後クラス

問1. 次の連立1次方程式を解きなさい。

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$$(1) \begin{cases} x - 2y - 2z = 4. \\ 3x - 5y - 7z = 11 \\ 2x - y - 7z = 5 \end{cases}, \quad (2) \begin{cases} x + 3y + z = 2 \\ 2x + y + z = -3 \\ -x + 2y + z = 7 \end{cases}$$

問2 次の行列の逆行列を求めなさい。

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$$(1) A = \begin{pmatrix} 1 & -1 & -3 \\ -1 & 2 & 5 \\ -1 & 1 & 4 \end{pmatrix}, \quad (2) B = \begin{pmatrix} 1 & 1 & -2 & -3 \\ 1 & 2 & -3 & -7 \\ -1 & -2 & 4 & 1 \\ 1 & 2 & -2 & -12 \end{pmatrix}$$

問3 次の行列式を求めなさい。

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$$(1) \begin{vmatrix} 1 & 4 & 1 & 6 \\ 1 & 6 & 6 & 9 \\ 2 & 6 & -6 & 7 \\ 2 & 6 & -6 & 5 \end{vmatrix}, \quad (2) \begin{vmatrix} 3 & 5 & 1 & 2 & -1 \\ 2 & 6 & 0 & 9 & 1 \\ 0 & 0 & 7 & 1 & 2 \\ 0 & 0 & 3 & 2 & 5 \\ 0 & 0 & 0 & 0 & -6 \end{vmatrix}$$

問4. 次の行列式を求めなさい。

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$$\begin{vmatrix} t+3 & -2 & 4 \\ -3 & t+1 & -3 \\ -5 & 2 & t-6 \end{vmatrix}$$

午後7行解答

①

問1.

(1) 拡大係数行列を変形

$$\left(\begin{array}{ccc|c} 1 & -2 & -2 & 4 \\ 3 & -5 & -7 & 11 \\ 2 & -1 & -7 & 5 \end{array} \right) \xrightarrow{\substack{\textcircled{2}-3\textcircled{1} \\ \textcircled{3}-2\textcircled{1}}} \left(\begin{array}{ccc|c} 1 & -2 & -2 & 4 \\ 0 & 1 & -1 & -1 \\ 0 & 3 & -3 & -3 \end{array} \right)$$

$$\xrightarrow{\substack{\textcircled{1}+2\textcircled{2} \\ \textcircled{3}-3\textcircled{2}}} \left(\begin{array}{ccc|c} 1 & 0 & -4 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right) \text{ したがって、} \begin{cases} x-4z=2 \\ y-z=-1 \end{cases} \text{ を表す。}$$

$$z = s \text{ とおけば } \begin{cases} x = 2 + 4s \\ y = -1 + s \\ z = s \end{cases}$$

$$(2) \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 2 & 1 & 1 & -3 \\ -1 & 2 & 1 & 7 \end{array} \right) \xrightarrow{\substack{\textcircled{2}-2\textcircled{1} \\ \textcircled{3}+\textcircled{1}}} \left(\begin{array}{ccc|c} 1 & 3 & 1 & 2 \\ 0 & -5 & -1 & -7 \\ 0 & 5 & 2 & 9 \end{array} \right)$$

$$\xrightarrow{\substack{\textcircled{2}+\textcircled{3} \\ \textcircled{1}-\frac{3}{5}\textcircled{2} \\ \textcircled{3}\times\frac{1}{5}}} \left(\begin{array}{ccc|c} 1 & 0 & -\frac{1}{5} & -\frac{17}{5} \\ 0 & 0 & 1 & 2 \\ 0 & 1 & \frac{2}{5} & \frac{9}{5} \end{array} \right) \xrightarrow{\substack{\textcircled{1}+\frac{1}{5}\textcircled{2} \\ \textcircled{3}-\frac{2}{5}\textcircled{2}}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{array} \right)$$

$$\begin{cases} x = -3 \\ y = 1 \\ z = 2 \end{cases}$$

問 2

(1)

$$(AE_3) = \left(\begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ -1 & 2 & 5 & 0 & 1 & 0 \\ -1 & 1 & 4 & 0 & 0 & 1 \end{array} \right)$$

$$\begin{array}{l} \xrightarrow{\textcircled{2} + \textcircled{1}} \\ \textcircled{3} + \textcircled{1} \end{array} \left(\begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array} \right)$$

$$\therefore A^{-1} = \begin{pmatrix} 3 & 1 & 1 \\ -1 & 1 & -2 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\textcircled{1} + \textcircled{2}} \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} \textcircled{1} + \textcircled{3} \\ \textcircled{2} - 2\textcircled{3} \end{array}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 1 \\ 0 & 1 & 0 & -1 & 1 & -2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array} \right) = (E_3 A^{-1})$$

(2)

$$(BE_4) = \left(\begin{array}{cccc|cccc} 1 & 1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 1 & 2 & -3 & -7 & 0 & 1 & 0 & 0 \\ -1 & -2 & 4 & 1 & 0 & 0 & 1 & 0 \\ 1 & 2 & -2 & -12 & 0 & 0 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} \textcircled{2} - \textcircled{1} \\ \textcircled{3} + \textcircled{1} \\ \textcircled{4} - \textcircled{1} \end{array}} \left(\begin{array}{cccc|cccc} 1 & 1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -4 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & -2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -9 & -1 & 0 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} \textcircled{1} - \textcircled{2} \\ \textcircled{3} + \textcircled{2} \\ \textcircled{4} - \textcircled{2} \end{array}} \left(\begin{array}{cccc|cccc} 1 & 0 & -1 & 1 & 2 & -1 & 0 & 0 \\ 0 & 1 & -1 & -4 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -5 & 0 & -1 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} \textcircled{1} + \textcircled{3} \\ \textcircled{2} + \textcircled{3} \\ \textcircled{4} - \textcircled{3} \end{array}} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -5 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & -10 & -1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -6 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -2 & -1 & 1 \end{array} \right)$$

$$\begin{matrix} \textcircled{1} + 5\textcircled{4} \\ \textcircled{2} + 10\textcircled{4} \\ \textcircled{3} + 6\textcircled{4} \end{matrix} \rightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -10 & -4 & 5 \\ 0 & 1 & 0 & 0 & -1 & -18 & -9 & 10 \\ 0 & 0 & 1 & 0 & 0 & -11 & -5 & 6 \\ 0 & 0 & 0 & 1 & 0 & -2 & -1 & 1 \end{array} \right) = (E_4 | B^{-1})$$

$$\therefore B^{-1} = \begin{pmatrix} 2 & -10 & -4 & 5 \\ -1 & -18 & -9 & 10 \\ 0 & -11 & -5 & 6 \\ 0 & -2 & -1 & 1 \end{pmatrix}$$

प्र. 3

(1)

$$\begin{vmatrix} 1 & 4 & 1 & 6 \\ 1 & 6 & 6 & 9 \\ 2 & 6 & -6 & 7 \\ 2 & 6 & -6 & 5 \end{vmatrix} \begin{matrix} \textcircled{2} - \textcircled{1} \\ \textcircled{3} - 2\textcircled{1} \\ \textcircled{4} - 2\textcircled{1} \end{matrix} = \begin{vmatrix} 1 & 4 & 1 & 6 \\ 0 & 2 & 5 & 3 \\ 0 & -2 & -8 & -5 \\ 0 & -2 & -8 & -7 \end{vmatrix} = 1 \begin{vmatrix} 2 & 5 & 3 \\ -2 & -8 & -5 \\ -2 & -8 & -7 \end{vmatrix}$$

$$= 2 \begin{vmatrix} 1 & 5 & 3 \\ -1 & -8 & -5 \\ -1 & -8 & -7 \end{vmatrix} \begin{matrix} \textcircled{2} + \textcircled{1} \\ \textcircled{3} + \textcircled{1} \end{matrix} = 2 \begin{vmatrix} 1 & 5 & 3 \\ 0 & -3 & -2 \\ 0 & -3 & -4 \end{vmatrix} = 2 \begin{vmatrix} -3 & -2 \\ -3 & -4 \end{vmatrix} = 2(12 - 6) = 12$$

(2)

$$\begin{vmatrix} 3 & 5 & 12 & -1 \\ 2 & 6 & 0 & 9 & 1 \\ 0 & 0 & 7 & 1 & 2 \\ 0 & 0 & 3 & 2 & 5 \\ 0 & 0 & 0 & 0 & -6 \end{vmatrix} = (-1)^{5+5} (-6) \begin{vmatrix} 3 & 5 & 12 \\ 2 & 6 & 0 & 9 \\ 0 & 0 & 7 & 1 \\ 0 & 0 & 3 & 2 \end{vmatrix}$$

$$\begin{matrix} \textcircled{3} - 7\textcircled{4} \end{matrix} \begin{vmatrix} 3 & 5 & -13 & 2 \\ 2 & 6 & -63 & 9 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -11 & 2 \end{vmatrix} = (-6) \times (-1)^{3+4} \begin{vmatrix} 3 & 5 & -13 \\ 2 & 6 & -63 \\ 0 & 0 & -11 \end{vmatrix}$$

$$= 6 \times (-11) \begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix} = -66 (18 - 10) = -528$$

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$$\begin{vmatrix} t+3 & -2 & 4 \\ -3 & t+1 & -3 \\ -5 & 2 & t-6 \end{vmatrix}$$

$$= (t+3) \begin{vmatrix} t+1 & -3 \\ 2 & t-6 \end{vmatrix} - (-3) \begin{vmatrix} -2 & 4 \\ 2 & t-6 \end{vmatrix} + (-5) \begin{vmatrix} -2 & 4 \\ t+1 & -3 \end{vmatrix}$$

$$= (t+3) \{ (t+1)(t-6) + 6 \} + 3(-2t+12-8) - 5(6-4t-4)$$

$$= (t+3)(t^2-5t) + 3(-2t+4) - 5(-4t+2)$$

$$= t^3 - 5t^2 + 3t^2 - 15t - 6t + 12 + 20t - 10$$

$$= t^3 - 2t^2 - t + 2 = \underline{(t+1)(t-1)(t-2)}$$