

期末試験・水曜・午前クラス

問1. 次の連立1次方程式を解きなさい。

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$$(1) \begin{cases} x + 2z = 1 \\ 2x + y + z = 0 \\ x - y + 2z = 0 \end{cases}, \quad (2) \begin{cases} x + 2z = 1 \\ 2x + y + 3z = 3 \\ x - y + 3z = 0 \end{cases}$$

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問2. 次の行列の逆行列を求めなさい。

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$$(1) A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & -1 \end{pmatrix}, \quad (2) B = \begin{pmatrix} 1 & 1 & -2 & -3 \\ 1 & 2 & -3 & -7 \\ -1 & -2 & 4 & 1 \\ 1 & 2 & -2 & -12 \end{pmatrix}$$

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問3. 次の行列式を求めなさい。

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$$(1) \begin{vmatrix} 3 & 1 & 1 & 5 \\ 0 & 1 & 0 & 1 \\ 1 & -1 & 1 & 3 \\ 2 & 0 & 1 & 2 \end{vmatrix}, \quad (2) \begin{vmatrix} 1 & 2 & 1 & 3 & 5 \\ 1 & 2 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 2 & 1 & 4 & 2 & 3 \\ 1 & 1 & 5 & 5 & 1 \end{vmatrix}$$

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問4. 次の行列式を求めなさい。

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$$\begin{vmatrix} t+3 & -2 & 4 \\ -3 & t+1 & -3 \\ -5 & 2 & t-6 \end{vmatrix}$$

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問1、

(1) 拡大係数行列を变形

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & -1 & 2 & 0 \end{array} \right) \xrightarrow{\substack{\textcircled{2}-2\textcircled{1} \\ \textcircled{3}-\textcircled{1}}} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -3 & -2 \\ 0 & -1 & 0 & -1 \end{array} \right)$$

$$\xrightarrow{\textcircled{2}+\textcircled{3}} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 0 & -3 & -3 \\ 0 & -1 & 0 & -1 \end{array} \right) \xrightarrow{\textcircled{1}+\textcircled{3} \left(\frac{2}{3}\right)} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 0 & -3 & -3 \\ 0 & -1 & 0 & -1 \end{array} \right)$$

$$\xrightarrow{\substack{\textcircled{2} \times (-\frac{1}{3}) \\ \textcircled{3} \times (-1)}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{array} \right) \quad \begin{cases} x = -1 \\ y = 1 \\ z = 1 \end{cases}$$

(2)

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 2 & 1 & 3 & 3 \\ 1 & -1 & 3 & 0 \end{array} \right) \xrightarrow{\substack{\textcircled{2}-2\textcircled{1} \\ \textcircled{3}-\textcircled{1}}} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 1 & -1 \end{array} \right)$$

$$\xrightarrow{\textcircled{3}+\textcircled{2}} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad \begin{cases} x+2z=1 \\ y-z=1 \end{cases}$$

$z = s$ とおけば、

$$\begin{cases} x = 1 - 2s \\ y = 1 + s \\ z = s \end{cases}$$

問2、

(2)

$$(1) (A E_3) = \left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{\textcircled{1} - \textcircled{2}} \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 \end{array} \right)$$

$$\therefore A^{-1} = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

$$\xrightarrow{\textcircled{3} + \frac{1}{2}\textcircled{1}} \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & \frac{1}{2} & 0 & \frac{1}{2} \end{array} \right)$$

$$\xrightarrow{\substack{\textcircled{1} \times \frac{1}{2} \\ \textcircled{3} \times (-1)}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \end{array} \right) = (E_3 : A^{-1})$$

$$(2) (B E_4) = \left(\begin{array}{cccc|cccc} 1 & 1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 1 & 2 & -3 & -7 & 0 & 1 & 0 & 0 \\ -1 & -2 & 4 & 1 & 0 & 0 & 1 & 0 \\ 1 & 2 & -2 & -12 & 0 & 0 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{\substack{\textcircled{2} - \textcircled{1} \\ \textcircled{3} + \textcircled{1} \\ \textcircled{4} - \textcircled{1}}} \left(\begin{array}{cccc|cccc} 1 & 1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -4 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & -2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -9 & -1 & 0 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{\substack{\textcircled{1} - \textcircled{2} \\ \textcircled{3} + \textcircled{2} \\ \textcircled{4} - \textcircled{2}}} \left(\begin{array}{cccc|cccc} 1 & 0 & -1 & 1 & 2 & -1 & 0 & 0 \\ 0 & 1 & -1 & -4 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -5 & 0 & -1 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{\substack{\textcircled{1} + \textcircled{3} \\ \textcircled{2} + \textcircled{3} \\ \textcircled{4} - \textcircled{3}}} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -5 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & -10 & -1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -6 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -2 & -1 & 1 \end{array} \right)$$

$$\begin{matrix} \textcircled{1} + 5\textcircled{4} \\ \textcircled{2} + 10\textcircled{4} \\ \textcircled{3} + 6\textcircled{4} \end{matrix} \rightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -10 & -4 & 5 \\ 0 & 1 & 0 & 0 & -1 & -18 & -9 & 10 \\ 0 & 0 & 1 & 0 & 0 & -11 & -5 & 6 \\ 0 & 0 & 0 & 1 & 0 & -2 & -1 & 1 \end{array} \right) = (E_4 | B^{-1})$$

$$\therefore B^{-1} = \begin{pmatrix} 2 & -10 & -4 & 5 \\ -1 & -18 & -9 & 10 \\ 0 & -11 & -5 & 6 \\ 0 & -2 & -1 & 1 \end{pmatrix}$$

問3.

$$(1) \left| \begin{array}{cccc|c} 3 & 1 & 1 & 5 & \\ 0 & 1 & 0 & 1 & \\ 1 & -1 & 1 & 3 & \\ 2 & 0 & 1 & 2 & \end{array} \right| \xrightarrow{\substack{\textcircled{1} + \textcircled{3} \\ \textcircled{2} + \textcircled{3}}} \left| \begin{array}{cccc|c} 4 & 0 & 2 & 8 & \\ 1 & 0 & 1 & 4 & \\ 1 & -1 & 1 & 3 & \\ 2 & 0 & 1 & 2 & \end{array} \right| \xrightarrow{(-1) \textcircled{2} (-1) \textcircled{3}} \left| \begin{array}{ccc|c} 4 & 2 & 8 & \\ 1 & 1 & 4 & \\ 2 & 1 & 2 & \end{array} \right|$$

$$= 2 \left| \begin{array}{ccc|c} 2 & 1 & 4 & \\ 1 & 1 & 4 & \\ 2 & 1 & 2 & \end{array} \right| \xrightarrow{\substack{\textcircled{1} - \textcircled{2} \\ \textcircled{3} - \textcircled{2}}} 2 \left| \begin{array}{ccc|c} 1 & 0 & 0 & \\ 1 & 1 & 4 & \\ 1 & 0 & -2 & \end{array} \right| = 2 \left| \begin{array}{cc|c} 1 & 0 & \\ 1 & -2 & \end{array} \right| = -4$$

$$(2) \left| \begin{array}{ccccc|c} 1 & 2 & 1 & 3 & 5 & \\ 1 & 2 & 0 & 0 & 2 & \\ 0 & 0 & -1 & 0 & 2 & \\ 2 & 1 & 4 & 2 & 3 & \\ 1 & 1 & 5 & 5 & 1 & \end{array} \right| \xrightarrow{\substack{\textcircled{2} - \textcircled{1} \\ \textcircled{4} - 2\textcircled{1} \\ \textcircled{5} - \textcircled{1}}} \left| \begin{array}{ccccc|c} 1 & 2 & 1 & 3 & 5 & \\ 0 & 0 & -1 & -3 & -3 & \\ 0 & 0 & -1 & 0 & 1 & \\ 0 & -3 & 2 & -4 & -7 & \\ 0 & -1 & 4 & 2 & -4 & \end{array} \right|$$

$$= 1 \left| \begin{array}{cccc|c} 0 & -1 & -3 & -3 & \\ 0 & 1 & 0 & -1 & \\ -3 & 2 & -4 & -7 & \\ -1 & 4 & 2 & -4 & \end{array} \right| \xrightarrow{\textcircled{2} - \textcircled{4}} \left| \begin{array}{cccc|c} 0 & 2 & -3 & -3 & \\ 0 & 0 & 0 & -1 & \\ -3 & 9 & -4 & -7 & \\ -1 & 8 & 2 & -4 & \end{array} \right|$$

$$= (-1)^{2+4} 1 \left| \begin{array}{ccc|c} 0 & 2 & -3 & \\ -3 & 9 & -4 & \\ -1 & 8 & 2 & \end{array} \right| \xrightarrow{\textcircled{2} - 3\textcircled{3}} \left| \begin{array}{ccc|c} 0 & 2 & -3 & \\ 0 & -15 & -10 & \\ -1 & 8 & 2 & \end{array} \right| = (-1)$$

$$= (-1)^{3+1} (-1) \begin{vmatrix} 2 & -3 \\ -15 & -10 \end{vmatrix} = 5 \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = 5(4+9) = \underline{65}$$

Pr. 4

$$\begin{vmatrix} t+3 & -2 & 4 \\ -3 & t+1 & -3 \\ -5 & 2 & t-6 \end{vmatrix}$$

$$= (t+3) \begin{vmatrix} t+1 & -3 \\ 2 & t-6 \end{vmatrix} - (-3) \begin{vmatrix} -2 & 4 \\ 2 & t-6 \end{vmatrix} + (-5) \begin{vmatrix} -2 & 4 \\ t+1 & -3 \end{vmatrix}$$

$$= (t+3) \{ (t+1)(t-6) + 6 \} + 3(-2t+12-8) - 5(6-4t-4)$$

$$= (t+3)(t^2-5t) + 3(-2t+4) - 5(-4t+2)$$

$$= t^3 - 5t^2 + 3t^2 - 15t - 6t + 12 + 20t - 10$$

$$= t^3 - 2t^2 - t + 2 = \underline{\underline{(t+1)(t-1)(t-2)}}$$