

問

次の行列 A の逆行列を求めよ。

$$A = \begin{bmatrix} 3 & 3 & -5 & -6 \\ 1 & 2 & -3 & -1 \\ 2 & 3 & -5 & -3 \\ -1 & 0 & 2 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 4 & \boxed{(1)} & \boxed{(2)} & -3 \\ 0 & -1 & 1 & 1 \\ 1 & 3 & -3 & 0 \\ 1 & 6 & -5 & -1 \end{bmatrix}$$

$(1) = 18$

$(2) = -16$

| | | | | | | | |
|----|---|----|----|---|---|---|---|
| 3 | 3 | -5 | -6 | 1 | | | |
| 1 | 2 | -3 | -1 | | 1 | | |
| 2 | 3 | -5 | -3 | | | 1 | |
| -1 | 0 | 2 | 2 | | | | 1 |

\downarrow
 $\textcircled{1} + 3 \times \textcircled{4}$
 $\textcircled{2} + \textcircled{4}, \textcircled{3} + 2 \times \textcircled{4}$

| | | | | | | | |
|----|---|----|---|---|---|---|---|
| 0 | 3 | 1 | 0 | 1 | 0 | 0 | 3 |
| 0 | 2 | -1 | 1 | 0 | 1 | 0 | 1 |
| 0 | 3 | -1 | 1 | 0 | 0 | 1 | 2 |
| -1 | 0 | 2 | 2 | 0 | 0 | 0 | 1 |

\downarrow
 $\textcircled{1} - \textcircled{3}$
 $\textcircled{2} - \textcircled{3}$

| | | | | | | | |
|----|----|----|----|---|---|----|----|
| 0 | 0 | 2 | -1 | 1 | 0 | -1 | 1 |
| 0 | -1 | 0 | 0 | 0 | 1 | -1 | -1 |
| 0 | 3 | -1 | 1 | 0 | 0 | 1 | 2 |
| -1 | 0 | 2 | 2 | 0 | 0 | 0 | 1 |

\downarrow
 $\textcircled{3} + 3 \times \textcircled{2}$

| | | | | | | | |
|----|----|----|----|---|---|----|----|
| 0 | 0 | 2 | -1 | 1 | 0 | -1 | 1 |
| 0 | -1 | 0 | 0 | 0 | 1 | -1 | -1 |
| 0 | 0 | -1 | 1 | 0 | 3 | -2 | -1 |
| -1 | 0 | 2 | 2 | 0 | 0 | 0 | 1 |

| | | | | | | | |
|---|---|---|---|---|----|-----|----|
| 1 | 0 | 0 | 0 | 4 | 18 | -16 | -3 |
| 0 | 1 | 0 | 0 | 0 | -1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 3 | -3 | 0 |
| 0 | 0 | 0 | 1 | 1 | 6 | -5 | -1 |

\uparrow

| | | | | | | | |
|---|---|---|---|---|----|-----|----|
| 0 | 0 | 1 | 0 | 1 | 3 | -3 | 0 |
| 0 | 1 | 0 | 0 | 0 | -1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 6 | -5 | -1 |
| 1 | 0 | 0 | 0 | 4 | 18 | -16 | -3 |

$\uparrow \textcircled{4} + 2 \times \textcircled{1} + 2 \times \textcircled{3}$

| | | | | | | | |
|---|---|----|----|---|----|----|----|
| 0 | 0 | 1 | 0 | 1 | 3 | -3 | 0 |
| 0 | 1 | 0 | 0 | 0 | -1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 6 | -5 | -1 |
| 1 | 0 | -2 | -2 | 0 | 0 | 0 | -1 |

\uparrow
 $\textcircled{3} + \textcircled{1}$
 $(-1) \times \textcircled{4}$

| | | | | | | | |
|----|---|----|---|---|----|----|----|
| 0 | 0 | 1 | 0 | 1 | 3 | -3 | 0 |
| 0 | 1 | 0 | 0 | 0 | -1 | 1 | 1 |
| 0 | 0 | -1 | 1 | 0 | 3 | -2 | -1 |
| -1 | 0 | 2 | 2 | 0 | 0 | 0 | 1 |



$\textcircled{1} + \textcircled{3}$
 $(-1) \times \textcircled{2}$