

数学C 2019 (2回目レポート)

①

問 次の行列の階数 (rank) を求めなさい。

$$A = \begin{pmatrix} 1 & 1 & 4 & 2 \\ 2 & -1 & -1 & 1 \\ -1 & 1 & 2 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 2 & 2 & -1 \\ 1 & 2 & 0 & 2 & 1 \\ -1 & 1 & 1 & 0 & 0 \\ 1 & 5 & 1 & 4 & 2 \end{pmatrix}$$

$$\text{rank } A = \boxed{(1)}$$

$$\text{rank } B = \boxed{(2)}$$

$\boxed{(1)} = 2 \quad \boxed{(2)} = 3$

$$A = \begin{pmatrix} 1 & 1 & 4 & 2 \\ 2 & -1 & -1 & 1 \\ -1 & 1 & 2 & 0 \end{pmatrix} \xrightarrow[\substack{\textcircled{2} - 2 \times \textcircled{1} \\ \textcircled{3} + \textcircled{1}}]{\quad} \begin{pmatrix} 1 & 1 & 4 & 2 \\ 0 & -3 & -9 & -3 \\ 0 & 2 & 6 & 2 \end{pmatrix}$$

$$\xrightarrow[\substack{\textcircled{2} \times (-\frac{1}{3}) \\ \textcircled{3} \times \frac{1}{2}}]{\quad} \begin{pmatrix} 1 & 1 & 4 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 1 & 3 & 1 \end{pmatrix} \xrightarrow[\substack{\textcircled{1} - \textcircled{2} \\ \textcircled{3} - \textcircled{2}}]{\quad} \begin{pmatrix} \boxed{1} & \boxed{0} & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

rank A = 2

$$B = \begin{pmatrix} 1 & 0 & 2 & 2 & -1 \\ 1 & 2 & 0 & 2 & 1 \\ -1 & 1 & 1 & 0 & 0 \\ -1 & 5 & 1 & 4 & 2 \end{pmatrix} \xrightarrow[\substack{\textcircled{2} - \textcircled{1} \\ \textcircled{3} + \textcircled{1} \\ \textcircled{4} - \textcircled{1}}]{\quad} \begin{pmatrix} 1 & 0 & 2 & 2 & -1 \\ 0 & 2 & -2 & 0 & 2 \\ 0 & 1 & 3 & 2 & -1 \\ 0 & 5 & -1 & 2 & 3 \end{pmatrix}$$

$$\xrightarrow[\substack{\textcircled{2} - 2 \times \textcircled{3} \\ \textcircled{4} - 5 \times \textcircled{3}}]{\quad} \begin{pmatrix} 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & -8 & -4 & 4 \\ 0 & 1 & 3 & 2 & -1 \\ 0 & 0 & -16 & -8 & 8 \end{pmatrix} \xrightarrow[\substack{\textcircled{2} \times (-\frac{1}{4}) \\ \textcircled{4} \times (-\frac{1}{8})}]{\quad} \begin{pmatrix} 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 3 & 2 & -1 \\ 0 & 0 & 2 & 1 & -1 \end{pmatrix}$$

$$\xrightarrow[\substack{\textcircled{1} - \textcircled{2} \\ \textcircled{4} - \textcircled{2} \\ \textcircled{3} - \frac{3}{2} \textcircled{2}}]{\quad} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow[\frac{1}{2} \times \textcircled{2}]{\quad} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow[\textcircled{2} \leftrightarrow \textcircled{3}]{\quad} \begin{pmatrix} \boxed{1} & \boxed{0} & \boxed{0} & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

rank B = 3