微積分解法2019 レポート (8回目)

問. 次の微分方程式を解け.

(1)

$$\frac{dy}{dx} = \frac{(1+x^2)y^3}{(y^2-1)x^3}$$

$$x^a + y^a = 2\log\left(C\left|\frac{x}{y}\right|\right)$$

$a =$ (1)

(2)

$$(1+y^2) + (1+x^2)\frac{dy}{dx} = 0$$

$$\mathrm{Arctan}\, x + b\,\mathrm{Arctan}\, y = C$$

$b =$ (2)

$\boxed{(1)} = -2$

$\boxed{(2)} = 1$

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(1)

$$\frac{y^2-1}{y^3}\frac{dy}{dx} = \frac{1+x^2}{x^3}$$

$$\int\left(\frac{1}{y}-\frac{1}{y^3}\right)dy = \int\left(\frac{1}{x^3}+\frac{1}{x}\right)dx$$

$$\log|y| + \frac{1}{2}y^{-2} = -\frac{1}{2}y^{-2} + \log|x| + C'$$

$$\log\left(e^{C'}\left|\frac{x}{y}\right|\right) = \frac{1}{2}(x^{-2}+y^{-2})$$

$$\therefore \underline{2\log\left(C\left|\frac{x}{y}\right|\right) = x^{-2}+y^{-2}}$$

(2)

$$\frac{1}{1+y^2}\frac{dy}{dx} = -\frac{1}{1+x^2}$$

$$\int\frac{1}{1+y^2}dy = -\int\frac{1}{1+x^2}dx$$

$$\mathrm{Arctan}\, y = -\mathrm{Arctan}\, x + C$$

$$\underline{\mathrm{Arctan}\, x + \mathrm{Arctan}\, y = C}$$