

①

微積分解法 2019 (3回目)

問1 次の微分を計算しなさい。

$$f(x) = \text{Arcsin}(2x\sqrt{1-x^2})$$

$$f'(x) = \boxed{(1)} \frac{(1-2x^2)}{|1-2x^2|} \frac{1}{\sqrt{1-x^2}}$$

問2 次の微分を計算しなさい。

$$g(x) = x^2 e^x$$

$$g^{(5)}(x) = (x^2 + \boxed{(2)}x + 20)e^x$$

2

$$\boxed{(1)} = 2$$

$$\boxed{(2)} = 10$$

(3)

問1.

$$\begin{aligned}
 f'(x) &= \frac{1}{\sqrt{1-(2x\sqrt{1-x^2})^2}} (2x\sqrt{1-x^2})' \\
 &= \frac{1}{\sqrt{1-4x^2(1-x^2)}} \left(2\sqrt{1-x^2} + 2x \frac{\frac{1}{2}(-2x)}{\sqrt{1-x^2}} \right) \\
 &= \frac{1}{\sqrt{1-4x^2+4x^4}} \left(2\sqrt{1-x^2} - \frac{2x^2}{\sqrt{1-x^2}} \right) \\
 &= \frac{2(1-2x^2)}{\sqrt{(1-2x^2)^2} \sqrt{1-x^2}} \\
 &= \frac{2(1-2x^2)}{|1-2x^2| \sqrt{1-x^2}}
 \end{aligned}$$

問2.

$$\begin{aligned}
 g'(x) &= 2xe^x + x^2e^x \\
 &= (x^2+2x)e^x
 \end{aligned}$$

$$\begin{aligned}
 g''(x) &= (2x+2)e^x + (x^2+2x)e^x \\
 &= (x^2+4x+2)e^x
 \end{aligned}$$

$$\begin{aligned}
 g^{(3)}(x) &= (2x+4)e^x + (x^2+4x+2)e^x \\
 &= (x^2+6x+6)e^x
 \end{aligned}$$

$$\begin{aligned}
 g^{(4)}(x) &= (2x+6)e^x + (x^2+6x+6)e^x \\
 &= (x^2+8x+12)e^x
 \end{aligned}$$

④

$$\begin{aligned}g^{(5)}(x) &= (2x + 8)e^x + (x^2 + 8x + 12)e^x \\ &= (x^2 + 10x + 20)e^x\end{aligned}$$