微積分解法2019 (3回目)

問1 次の微分を計算しなさい。

$f(x) = \mathrm{Arcsin}\left(2x\sqrt{1-x^2}\right)$

$f'(x) =$ [11] $\dfrac{(1-2x^2)}{|1-2x^2|}\dfrac{1}{\sqrt{1-x^2}}$

問2 次の微分を計算しなさい。

$g(x) = x^2 e^x$

$g^{(5)}(x) = (x^2 +$ [12] $x + 20)e^x$

[(1)] = 2

[(2)] = 10

問1.

$$f'(x) = \frac{1}{\sqrt{1-(2x\sqrt{1-x^2})^2}} \left(2x\sqrt{1-x^2}\right)'$$

$$= \frac{1}{\sqrt{1-4x^2(1-x^2)}} \left(2\sqrt{1-x^2} + 2x \frac{\frac{1}{2}(-2x)}{\sqrt{1-x^2}}\right)$$

$$= \frac{1}{\sqrt{1-4x^2+4x^4}} \left(2\sqrt{1-x^2} - \frac{2x^2}{\sqrt{1-x^2}}\right)$$

$$= \frac{2(1-2x^2)}{\sqrt{(1-2x^2)^2}\sqrt{1-x^2}}$$

$$= \frac{2(1-2x^2)}{|1-2x^2|\sqrt{1-x^2}}$$

問2.

$$g'(x) = 2xe^x + x^2e^x$$

$$= (x^2+2x)e^x$$

$$g''(x) = (2x+2)e^x + (x^2+2x)e^x$$

$$= (x^2+4x+2)e^x$$

$$g^{(3)}(x) = (2x+4)e^x + (x^2+4x+2)e^x$$

$$= (x^2+6x+6)e^x$$

$$g^{(4)}(x) = (2x+6)e^x + (x^2+6x+6)e^x$$

$$= (x^2+8x+12)e^x$$

$$g^{(5)}(x) = (2x+8)e^x + (x^2+8x+12)e^x$$
$$= (x^2+10x+20)e^x$$