

# 数学I レポート (7回目)

問1. 次の積分を求めなさい。

$$(1) \quad f(x, y) = 1$$

$$D = \{(x, y) \mid 0 \leq y \leq 1, 0 \leq x \leq 4 - y\}$$

$$\iint_D f(x, y) dx dy = \frac{\boxed{(1)}}{2}$$

$$(2) \quad f(x, y) = xy$$

$$D = \{(x, y) \mid -1 \leq y \leq 1, 0 \leq x \leq 2 - y\}$$

$$\iint_D f(x, y) dx dy = -\frac{4}{\boxed{(2)}}$$

問2. 次の累次積分の積分の順序を交換しなさい。

$$\int_1^e \left( \int_0^{\log x} f(x, y) dy \right) dx$$

$$= \int_{\boxed{(4)}}^{\boxed{(3)}} \left( \int_{e^y}^e f(x, y) dx \right) dy$$

(1) = 7

(2) = 3

(3) = 1

(4) = 0

(i)

$$\begin{aligned}
 & \iint_D f(x, y) \, dx \, dy \\
 &= \int_0^1 \left( \int_0^{4-y} 1 \, dx \right) dy \\
 &= \int_0^1 [x]_{x=0}^{x=4-y} dy \\
 &= \int_0^1 (4-y) dy \\
 &= \left[ 4y - \frac{1}{2}y^2 \right]_{y=0}^{y=1} \\
 &= \frac{7}{2}
 \end{aligned}$$

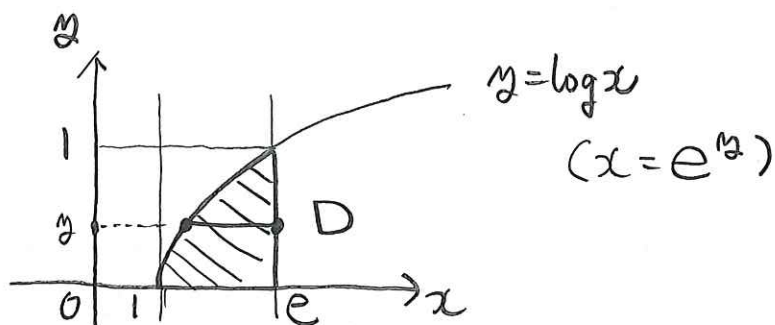
(ii)

$$\begin{aligned}
 & \iint_D f(x, y) \, dx \, dy \\
 &= \int_{-1}^1 \left( \int_0^{2-y} x^2 \, dx \right) dy \\
 &= \int_{-1}^1 \left[ \frac{1}{2}yx^2 \right]_{x=0}^{x=2-y} dy \\
 &= \int_{-1}^1 \frac{1}{2}(2-y)^2 y \, dy \\
 &= \int_{-1}^1 \left( \frac{1}{2}y^3 - 2y^2 + 2y \right) dy \\
 &= \left[ \frac{1}{8}y^4 - \frac{2}{3}y^3 + y^2 \right]_{y=-1}^{y=1} \\
 &= -\frac{4}{3}
 \end{aligned}$$



$$\int_1^e \left( \int_0^{\log x} f(x, y) dy \right) dx$$

積分領域を图示すると、



$$\begin{aligned} D &= \{ (x, y) \mid 1 \leq x \leq e, 0 \leq y \leq \log x \} \\ &= \{ (x, y) \mid 0 \leq y \leq 1, e^y \leq x \leq e \} \end{aligned}$$

$$\int_0^1 \left( \int_{e^y}^e f(x, y) dx \right) dy$$