

数学I 2019 第5回目

問1 $f(x, y) = C^2$ 級

$$\begin{cases} x(t) = a + ut \\ y(t) = b + vt \end{cases}$$

$z(t) = f(x(t), y(t))$ とする。

$$\frac{d^2 z}{dt^2} = u^2 f_{x,x} + \boxed{(1)} uv f_{x,y} + v^2 f_{y,y}$$

問2. $f(x, y) = \log(x^2 + y^2)$

$g(x, y) = \frac{x-y}{x+y}$ とする。

$$\begin{aligned} & \left(h \frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right)^2 f(x, y) \\ &= \left(h^2 \frac{\partial^2}{\partial x^2} + 2hk \frac{\partial^2}{\partial x \partial y} + k^2 \frac{\partial^2}{\partial y^2} \right) f(x, y) \\ &= -2h^2 \frac{x^2 - y^2}{(x^2 + y^2)^2} + 2hk \frac{\boxed{(2)} xy}{(x^2 + y^2)^2} \\ & \quad + k^2 \times \boxed{(3)} \frac{(x^2 - y^2)}{(x^2 + y^2)^2} \end{aligned}$$

$$\begin{aligned} & \left(h \frac{\partial}{\partial x} + k \frac{\partial}{\partial y} \right)^2 g(x, y) \\ &= \left(h^2 \frac{\partial^2}{\partial x^2} + 2hk \frac{\partial^2}{\partial x \partial y} + k^2 \frac{\partial^2}{\partial y^2} \right) g(x, y) \\ &= \boxed{(4)} \quad h^2 \frac{y}{(x+y)^3} + 4hk \frac{x-y}{(x+y)^3} + 4k^2 \frac{x}{(x+y)^3} \end{aligned}$$

$$\boxed{(1)} = 2$$

$$\boxed{(2)} = -4$$

$$\boxed{(3)} = 2$$

$$\boxed{(4)} = -4$$

問1.

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} u + \frac{\partial f}{\partial y} v$$

$$\frac{d^2z}{dt^2} = \left(\frac{\partial^2 f}{\partial x^2} u + \frac{\partial^2 f}{\partial y \partial x} v \right) \cdot u$$

$$+ \left(\frac{\partial^2 f}{\partial x \partial y} u + \frac{\partial^2 f}{\partial y^2} v \right) \cdot v$$

$$= u^2 \frac{\partial^2 f}{\partial x^2} + 2uv \frac{\partial^2 f}{\partial x \partial y} + v^2 \frac{\partial^2 f}{\partial y^2}$$

問2.

$$\boxed{(1)} = 2$$

$$\frac{\partial f}{\partial y}(x, y) = \frac{2y}{x^2 + y^2}$$

$$\frac{\partial^2 f}{\partial x \partial y}(x, y) = \frac{2y}{(x^2 + y^2)^2} (-1)(2x)$$

$$= -\frac{4xy}{(x^2 + y^2)^2}$$

$$\boxed{(2)} = -4$$

$$\frac{\partial^2 f}{\partial y^2}(x, y) = \frac{2(x^2 - y^2) - 2y \times 2y}{(x^2 + y^2)^2}$$

$$= \frac{2(x^2 - y^2)}{(x^2 + y^2)^2}$$

$$\boxed{(3)} = 2$$

$$\frac{\partial g}{\partial x} = \frac{+(x+y)-(x-y)}{(x+y)^2}$$

$$= \frac{+2y}{(x+y)^2}$$

$$\frac{\partial^2 g}{\partial x^2} = \frac{(+2y) \cdot -2}{(x+y)^3}$$

$$= \frac{-4y}{(x+y)^3}$$

$$\boxed{(4)} = -4$$