

数学I 2019 L110-ト4回目

問1

$z = f(x, y) = C^1$ 級

$x = u \cos \alpha - v \sin \alpha$

$y = u \sin \alpha + v \cos \alpha$ のとき,

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \boxed{(1)} \left(\frac{\partial z}{\partial u}\right)^2 + \boxed{(2)} \frac{\partial z}{\partial u} \cdot \frac{\partial z}{\partial v} + \boxed{(3)} \left(\frac{\partial z}{\partial v}\right)^2$$

$\boxed{(1)}, \boxed{(2)}, \boxed{(3)} = ?$

問2

$z = f(x, y) = C^1$ 級

$x = r \cos \theta$

$y = r \sin \theta$ のとき,

$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = \boxed{(4)} \cdot r \frac{\partial z}{\partial r}$

$\boxed{(4)} = ?$

$$(1) = 1$$

$$(2) = 0$$

$$(3) = 1$$

$$(4) = 1$$

3

問1.

$$\frac{\partial Z}{\partial u} = \cos \alpha \frac{\partial Z}{\partial x} + \sin \alpha \frac{\partial Z}{\partial y}$$

$$\frac{\partial Z}{\partial v} = -\sin \alpha \frac{\partial Z}{\partial x} + \cos \alpha \frac{\partial Z}{\partial y} \quad \text{である.}$$

$$\left(\frac{\partial Z}{\partial u}\right)^2 + \left(\frac{\partial Z}{\partial v}\right)^2$$

$$= \cos^2 \alpha \left(\frac{\partial Z}{\partial x}\right)^2 + 2\cos \alpha \sin \alpha \frac{\partial Z}{\partial x} \frac{\partial Z}{\partial y} + \sin^2 \alpha \left(\frac{\partial Z}{\partial y}\right)^2$$

$$+ \sin^2 \alpha \left(\frac{\partial Z}{\partial x}\right)^2 - 2\cos \alpha \sin \alpha \frac{\partial Z}{\partial x} \frac{\partial Z}{\partial y} + \cos^2 \alpha \left(\frac{\partial Z}{\partial y}\right)^2$$

$$= (\cos^2 \alpha + \sin^2 \alpha) \left(\left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2 \right)$$

$$= \left(\frac{\partial Z}{\partial x}\right)^2 + \left(\frac{\partial Z}{\partial y}\right)^2$$

問2.

$$\frac{\partial Z}{\partial r} = \frac{\partial Z}{\partial x} \cos \theta + \frac{\partial Z}{\partial y} \sin \theta$$

$$r \frac{\partial Z}{\partial r} = \frac{\partial Z}{\partial x} r \cos \theta + \frac{\partial Z}{\partial y} r \sin \theta$$

$$= x \frac{\partial Z}{\partial x} + y \frac{\partial Z}{\partial y}$$