

数学 I 2019 4月-1 2回目.

問1. $f(x,y) = y \log x + xy^3$ とする。

$$f_{x,y} = f_{y,x} = \frac{1}{x} + \boxed{(1)} y^2$$

問2. $g(x,y) = \log(x^2+y^2)$

$$g_x = \boxed{(2)} \times \frac{x}{x^2+y^2}$$

$$g_y = \boxed{(2)} \times \frac{y}{x^2+y^2}$$

$$g_{x,y} = g_{y,x} = \boxed{(3)} \times \frac{xy}{(x^2+y^2)^2}$$

問3. $f(x,y) = \begin{cases} \frac{x^3y}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

$$\frac{\partial^2 f}{\partial x \partial y}(0,0) = \underline{1}$$

$$\frac{\partial^2 f}{\partial y \partial x}(0,0) = \boxed{(4)}$$

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$$\boxed{(1)} = 3$$

$$\boxed{(2)} = 2$$

$$\boxed{(3)} = -4$$

$$\boxed{(4)} = 0.$$

問1.

$$f_x = \frac{1}{x} + y^3$$

$$f_y = \log x + 3xy^2$$

$$f_{xy} = \frac{1}{x} + 3y^2$$

$$f_{yx} = \frac{1}{x} + 3y^2$$

問2.

$$g_x = \frac{2xy}{x^2+y^2}$$

$$g_y = \frac{2y}{x^2+y^2}$$

$$g_{xy} = \frac{4xy}{(x^2+y^2)^2}$$

$$g_{yx} = \frac{4xy}{(x^2+y^2)^2}$$

問3.

$$f_x(x,y) = \begin{cases} \frac{x^4y + 3x^2y^3}{(x^2+y^2)^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$f_y(x,y) = \begin{cases} \frac{x^5 - x^3y^2}{(x^2+y^2)^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

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$$f_{x,y}(0,0) = \lim_{\Delta y \rightarrow 0} \frac{f_x(0, \Delta y) - f_x(0,0)}{\Delta y}$$

$$= \lim_{\Delta y \rightarrow 0} \frac{0 - 0}{\Delta y} = 0$$

$$f_{y,x}(0,0) = \lim_{\Delta x \rightarrow 0} \frac{f_y(\Delta x, 0) - f_y(0,0)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\Delta x^5}{\Delta x \Delta x^4} = 1$$