数学Ｃ・中間試験問題（平成３１年２月１日金曜クラス、７２名）

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問１．次の連立１次方程式系を解け。

(1) $\begin{cases} 2x-4y-z=3 \\ x-2y-z=2 \\ 3x-6y-2z=5 \end{cases}$, (2) $\begin{cases} x+2y-z+w=0 \\ x+y-2z-3w=0 \\ x+2y-3z-w=-4 \\ 2x-3y+z+w=3 \end{cases}$.

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問２．次の行列式の値を求めなさい。

(1) $\begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$, (2) $\begin{vmatrix} 1 & 1 & -1 & 2 \\ 1 & 3 & 3 & 2 \\ 1 & 1 & 1 & 2 \\ 0 & 2 & 3 & -1 \end{vmatrix}$.

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問３．次の行列 A, B の逆行列 A^{-1}, B^{-1} を求めなさい。

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 2 & 2 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 0 & -1 \\ -3 & -5 & 1 & 2 \\ 1 & 3 & 2 & -2 \\ 0 & 2 & 1 & -1 \end{pmatrix}.$$

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問４．行列 $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$ に関する以下の問いに答えなさい。

（１）行列式 $|A|$ を求めよ。 5

（２）行列 A の余因子行列 $\widetilde{A}$ を求めよ。 10

（３）（１）、（２）を用いて A の逆行列 A^{-1} を求めよ。 5

数C 2018 （全クラス）期末解答

(1) 拡大係数行列を基本変形する。

$$\left(\begin{array}{ccc|c} 2 & -4 & -1 & 3 \\ 1 & -2 & -1 & 2 \\ 3 & -6 & -2 & 5 \end{array}\right) \xrightarrow[\substack{①-2②\\③-3②}]{} \left(\begin{array}{ccc|c} 0 & 0 & 1 & -1 \\ 1 & -2 & -1 & 2 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

$$\xrightarrow[\substack{②+①\\③-①}]{} \left(\begin{array}{ccc|c} 0 & 0 & 1 & -1 \\ 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

つまり $z=-1$

$x-2y=1$

$y=t$ とおくと，

$$\begin{cases} x = 2t+1 \\ y = t \\ z = -1 \end{cases}$$ （t は任意）

(2) $$\left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 0 \\ 1 & 1 & -2 & -3 & 0 \\ 1 & 2 & -3 & -1 & -4 \\ 2 & -3 & 1 & 1 & 3 \end{array}\right)$$

$$\xrightarrow[\substack{②-①\\③-①\\④-2①}]{} \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 0 \\ 0 & -1 & -1 & -4 & 0 \\ 0 & 0 & -2 & -2 & -4 \\ 0 & -7 & 3 & -1 & 3 \end{array}\right)$$

$$\xrightarrow[\substack{\text{①}+2\text{②}\\ \text{④}-7\text{②}}]{} \left(\begin{array}{cccc|c} 1 & 0 & -3 & -7 & 0 \\ 0 & -1 & -1 & -4 & 0 \\ 0 & 0 & -2 & -2 & -4 \\ 0 & 0 & 10 & 27 & 3 \end{array}\right)$$

$$\xrightarrow[\substack{\text{②}\times(-1)\\ \text{③}\times(-\frac{1}{2})}]{} \left(\begin{array}{cccc|c} 1 & 0 & -3 & -7 & 0 \\ 0 & 1 & 1 & 4 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 10 & 27 & 3 \end{array}\right)$$

$$\xrightarrow[\substack{\text{①}+3\text{③}\\ \text{②}-\text{③}\\ \text{④}-10\text{③}}]{} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -4 & 6 \\ 0 & 1 & 0 & 3 & -2 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 17 & -17 \end{array}\right)$$

$$\xrightarrow[\text{④}\frac{1}{17}]{} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -4 & 6 \\ 0 & 1 & 0 & 3 & -2 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right)$$

$$\xrightarrow[\substack{\text{①}+4\text{④}\\ \text{②}-3\text{④}\\ \text{③}-\text{④}}]{} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right)$$

よって $\begin{cases} x = 2 \\ y = -1 \\ z = 3 \\ w = -1 \end{cases}$

問2.

(1)
$$\begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} \underset{③+②}{=\!=} \begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{vmatrix}$$

$$= (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} = 3-6 = \underline{-3}$$

(2)
$$\begin{vmatrix} 1 & 1 & -1 & 2 \\ 1 & 3 & 3 & 2 \\ 1 & 1 & 1 & 2 \\ 0 & 2 & 3 & -1 \end{vmatrix} \underset{\substack{②-① \\ ③-①}}{=\!=} \begin{vmatrix} 1 & 1 & -1 & 2 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 3 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 4 & 0 \\ 0 & 2 & 0 \\ 2 & 3 & -1 \end{vmatrix} = 2(-1)^{2+2} \begin{vmatrix} 2 & 0 \\ 2 & -1 \end{vmatrix}$$

$$= 2(-2) = \underline{-4}$$

問3

$$(A \vdots E_3) = \left(\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & & \\ -2 & -3 & -4 & & 1 & \\ 2 & 2 & 4 & & & 1 \end{array}\right)$$

$$\xrightarrow[②+③]{} \left(\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 \\ 2 & 2 & 4 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[③-2①]{} \left(\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 1 \\ 0 & -2 & -2 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[\substack{①+2② \\ ③-2②}]{} \left(\begin{array}{ccc:ccc} 1 & 0 & 3 & 1 & 2 & 2 \\ 0 & -1 & 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & -2 & -2 & -1 \end{array}\right)$$

$$\xrightarrow[\substack{②\times(-1) \\ ③\times(-\frac{1}{2})}]{} \left(\begin{array}{ccc:ccc} 1 & 0 & 3 & 1 & 2 & 2 \\ 0 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & 1 & \frac{1}{2} \end{array}\right)$$

$$\xrightarrow[①-3③]{} \left(\begin{array}{ccc:ccc} 1 & 0 & 0 & -2 & -1 & +\frac{1}{2} \\ 0 & 1 & 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & 1 & \frac{1}{2} \end{array}\right) = (E_3 \vdots A^{-1})$$

$$\therefore A^{-1} = \begin{pmatrix} -2 & -1 & \frac{1}{2} \\ 0 & -1 & -1 \\ 1 & 1 & \frac{1}{2} \end{pmatrix}$$

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$$(B \,\vdots\, E_4) = \left(\begin{array}{cccc:cccc} 1 & 2 & 0 & -1 & 1 & & & \\ -3 & -5 & 1 & 2 & & 1 & & \\ 1 & 3 & 2 & -2 & & & 1 & \\ 0 & 2 & 1 & -1 & & & & 1 \end{array}\right)$$

$$\xrightarrow[\substack{②+3① \\ ③-①}]{} \left(\begin{array}{cccc:cccc} 1 & 2 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & -1 & 0 & 1 & 0 \\ 0 & 2 & 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[①-④]{} \left(\begin{array}{cccc:cccc} 1 & 0 & -1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & -1 & 0 & 1 & 0 \\ 0 & 2 & 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[\substack{③-② \\ ④-2②}]{} \left(\begin{array}{cccc:cccc} 1 & 0 & -1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -4 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -6 & -2 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[\substack{①+③ \\ ②-③ \\ ④+③}]{} \left(\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & -3 & -1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 7 & 2 & -1 & 0 \\ 0 & 0 & 1 & 0 & -4 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -10 & -3 & 1 & 1 \end{array}\right)$$

$$\xrightarrow[②+④]{} \left(\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & -3 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 & -3 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & -4 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -10 & -3 & 1 & 1 \end{array}\right) = (E_4 \,\vdots\, B^{-1})$$

$$\therefore\ B^{-1} = \begin{pmatrix} -3 & -1 & 1 & -1 \\ -3 & -1 & 0 & 1 \\ -4 & -1 & 1 & 0 \\ -10 & -3 & 1 & 1 \end{pmatrix}$$

問4.

(1)

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix} \underset{\substack{②-2① \\ ③-3①}}{=\!=\!=} \begin{vmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & -5 & -7 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & -5 \\ -5 & -7 \end{vmatrix} = 7-25 = \underline{-18}$$

(2) A_{ij} を i行 j列をのぞいた行列とする.

$$|A_{11}| = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 6-1 = 5$$

$$|A_{12}| = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = 4-3 = 1$$

$$|A_{13}| = \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = 2-9 = -7$$

$$|A_{21}| = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = 1$$

$$|A_{22}| = \begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} = -7$$

$$|A_{23}| = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -5$$

$$|A_{31}| = \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} = -7$$

$$|A_{32}| = \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = -5$$

$$|A_{33}| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$$

$$\tilde{A} = \begin{pmatrix} |A_{11}| & -|A_{21}| & |A_{31}| \\ -|A_{12}| & |A_{22}| & -|A_{32}| \\ |A_{13}| & -|A_{23}| & |A_{33}| \end{pmatrix}$$

$$= \begin{pmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{pmatrix}$$

(3)

$$A^{-1} = \frac{1}{|A|}\tilde{A} = \frac{1}{18}\begin{pmatrix} -5 & 1 & 7 \\ 1 & 7 & -5 \\ 7 & -5 & 1 \end{pmatrix}$$