

数学C・期末試験問題（午後クラス52名、平成29年1月31日）

問1. 次の行列の行列式を求めよ。

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$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & 7 \\ 2 & 5 & -1 \\ 3 & 2 & 6 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 4 & 1 & 6 \\ 1 & 6 & 6 & 9 \\ 2 & 6 & -6 & 7 \\ 2 & 6 & -6 & 5 \end{pmatrix}.$$

問2. 次の行列の行列式を求めなさい。

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$$A = \begin{pmatrix} t-1 & 1 & 0 \\ -5 & t-6 & 5 \\ 1 & 2 & t+3 \end{pmatrix}.$$

問3. 次の行列の逆行列を求めなさい。

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$$A = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 & -3 \\ -1 & 2 & 5 \\ -1 & 1 & 4 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 & -2 & -3 \\ 1 & 2 & -3 & -7 \\ -1 & -2 & 4 & 1 \\ 1 & 2 & -2 & -12 \end{pmatrix}.$$

問4. 3次正方行列  $A$  を

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$$A = \begin{pmatrix} 1 & 3 & -3 \\ 3 & 5 & -6 \\ -1 & -2 & 2 \end{pmatrix}.$$

とする。

(1) 行列  $A$  の行列式を求めよ。 5

(2)  $n$  次正方行列  $B$  に対し  $B$  の第  $i$  行と第  $j$  列を除いて得られる  $(n-1)$  次正方行列を  $B_{i,j}$  とする。2次正方行列の行列式  $|A_{1,2}|, |A_{3,3}|$  を求めよ。

(3)  $A$  の余因子行列  $\tilde{A}$  を求めよ。 5 5

(4)  $A$  の逆行列  $A^{-1}$  を求めよ。 5

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問1.

$$\cdot |A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = \underline{-2}$$

$$|B| = \begin{vmatrix} 1 & 0 & 7 \\ 2 & 5 & -1 \\ 3 & 2 & 6 \end{vmatrix} \xrightarrow{(3\text{列}) - 7(1\text{列})} \begin{vmatrix} 1 & 0 & 0 \\ 2 & 5 & -15 \\ 3 & 2 & -15 \end{vmatrix}$$

1行を軸に展開.

$$(-1)^{1+1} 1 \times \begin{vmatrix} 5 & -15 \\ 2 & -15 \end{vmatrix} = 15(-5+2) = \underline{-45}$$

$$|C| = \begin{vmatrix} 1 & 4 & 1 & 6 \\ 1 & 6 & 6 & 9 \\ 2 & 6 & -6 & 7 \\ 2 & 6 & -6 & 5 \end{vmatrix} \xrightarrow{(4\text{行}) - (3\text{行})} \begin{vmatrix} 1 & 4 & 1 & 6 \\ 1 & 6 & 6 & 9 \\ 2 & 6 & -6 & 7 \\ 0 & 0 & 0 & -2 \end{vmatrix}$$

4行を軸に展開.

$$(-1)^{4+4} (-2) \begin{vmatrix} 1 & 4 & 1 \\ 1 & 6 & 6 \\ 2 & 6 & -6 \end{vmatrix} \xrightarrow{\begin{matrix} (2\text{行}) - (1\text{行}) \\ (3\text{行}) - 2(1\text{行}) \end{matrix}} (-2) \begin{vmatrix} 1 & 4 & 1 \\ 0 & 2 & 5 \\ 0 & -2 & -8 \end{vmatrix}$$

1列を軸に展開

$$(-2)(-1)^{4+1} 1 \begin{vmatrix} 2 & 5 \\ -2 & -8 \end{vmatrix} = (-2)(-16+10) = \underline{12}$$

問2.

$$|A| = \begin{vmatrix} t-1 & 1 & 0 \\ -5 & t-6 & 5 \\ 1 & 2 & t+3 \end{vmatrix}$$

1行之展開

$$(-1)^{11}(t-1) \begin{vmatrix} t-6 & 5 \\ 2 & t+3 \end{vmatrix} + (-1)^{12} \begin{vmatrix} -5 & 5 \\ 1 & t+3 \end{vmatrix}$$

$$= (t-1) \{ (t-6)(t+3) - 10 \} - \{ -5(t+3) - 5 \}$$

$$= (t-1)(t^2 - 3t - 28) + 5(t+4)$$

$$= (t-1)(t+4)(t-7) + 5(t+4)$$

$$= (t+4)(t^2 - 8t + 7 + 5)$$

$$= (t+4)(t^2 - 8t + 12)$$

$$= \underline{(t+4)(t-2)(t-6)}$$

問3.

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}}}$$

$$(B|E_3) = \left( \begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ -1 & 2 & 5 & 0 & 1 & 0 \\ -1 & 1 & 4 & 0 & 0 & 1 \end{array} \right)$$

$$\begin{array}{l} \xrightarrow{\text{②} + \text{①}} \\ \text{③} + \text{①} \end{array} \left( \begin{array}{ccc|ccc} 1 & -1 & -3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array} \right)$$

$$\begin{array}{l} \xrightarrow{\text{①} + 3 \times \text{③}} \\ \text{②} - 2 \times \text{③} \end{array} \left( \begin{array}{ccc|ccc} 1 & -1 & 0 & 4 & 0 & 3 \\ 0 & 1 & 0 & -1 & 1 & -2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{\textcircled{1} + \textcircled{2}} \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 1 \\ 0 & 1 & 0 & -1 & 1 & -2 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array} \right)$$

$$B^{-1} = \left( \begin{array}{ccc|ccc} 3 & 1 & 1 \\ -1 & 1 & -2 \\ 1 & 0 & 1 \end{array} \right)$$

$$(C; E_4) = \left[ \begin{array}{cccc|cccc} 1 & 1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 1 & 2 & -3 & -7 & 0 & 1 & 0 & 0 \\ -1 & -2 & 4 & 1 & 0 & 0 & 1 & 0 \\ 1 & 2 & -2 & -12 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\begin{array}{l} \textcircled{2} - \textcircled{1} \\ \textcircled{3} + \textcircled{1} \\ \textcircled{4} - \textcircled{1} \end{array}} \left[ \begin{array}{cccc|cccc} 1 & 1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -4 & -1 & 1 & 0 & 0 \\ 0 & -1 & 2 & -2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -9 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\begin{array}{l} \textcircled{1} - \textcircled{2} \\ \textcircled{3} + \textcircled{2} \\ \textcircled{4} - \textcircled{2} \end{array}} \left[ \begin{array}{cccc|cccc} 1 & 0 & -1 & 1 & 2 & -1 & 0 & 0 \\ 0 & 1 & -1 & -4 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -6 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -5 & 0 & -1 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\begin{array}{l} \textcircled{1} + \textcircled{3} \\ \textcircled{2} + \textcircled{3} \\ \textcircled{4} - \textcircled{3} \end{array}} \left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & -5 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & -10 & -1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -6 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -2 & -1 & 1 \end{array} \right]$$

$$\begin{array}{l} \text{①} + 5 \text{④} \\ \text{②} + 10 \text{④} \\ \text{③} + 6 \text{④} \end{array} \rightarrow \left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & -10 & -4 & 5 \\ 0 & 1 & 0 & 0 & -1 & -18 & -9 & 10 \\ 0 & 0 & 1 & 0 & 0 & -11 & -5 & 6 \\ 0 & 0 & 0 & 1 & 0 & -2 & -1 & 1 \end{array} \right]$$

$$C^{-1} = \begin{pmatrix} 2 & -10 & -4 & 5 \\ -1 & -18 & -9 & 10 \\ 0 & -11 & -5 & 6 \\ 0 & -2 & -1 & 1 \end{pmatrix}$$

問5.

$$(1) |A| = \begin{vmatrix} 1 & 3 & -3 \\ 3 & 5 & -6 \\ -1 & -2 & 2 \end{vmatrix} \xrightarrow{\substack{(2\text{行}) - 3(1\text{行}) \\ (3\text{行}) + (1\text{行})}} \begin{vmatrix} 1 & 3 & -3 \\ 0 & -4 & 3 \\ 0 & 1 & -1 \end{vmatrix}$$

1列を軸に展開

$$(-1)^{11} \cdot 1 \cdot \begin{vmatrix} -4 & 3 \\ 1 & -1 \end{vmatrix} = 4 - 3 = \underline{1}$$

$$(2) |A_{12}| = \begin{vmatrix} 3 & -6 \\ -1 & 2 \end{vmatrix} = 6 - 6 = \underline{0}$$

$$|A_{33}| = \begin{vmatrix} 1 & 3 \\ 3 & 5 \end{vmatrix} = 5 - 9 = \underline{-4}$$

$$(3) \hat{A} = \begin{pmatrix} -2 & 0 & -3 \\ 0 & -1 & -3 \\ -1 & -1 & -4 \end{pmatrix}$$

$$(4) A^{-1} = \frac{1}{|A|} \hat{A} = \begin{pmatrix} -2 & 0 & -3 \\ 0 & -1 & -3 \\ -1 & -1 & -4 \end{pmatrix}$$