

数学C・中間試験問題（午後クラス52名、平成29年12月20日）

20 問1. (1) $\frac{1+\sqrt{3}i}{2}$ を極形式で表せ。 10

(2) $(\frac{1+\sqrt{3}i}{2})^{-6}$ をもとめよ。 10

15 問2. 行列 $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$ に対して次を求めよ。

(1) $A+B$ 5

(2) $2A-3C$ 5

(3) ABC 5

15 問3. 行列 $A = \begin{pmatrix} 6 & 6 \\ -2 & -1 \end{pmatrix}$, $P = \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix}$ に対して次を求めよ。

(1) P^{-1} 5

(2) $P^{-1}AP$ 5

(3) A^n ($n=1,2,3,\dots$) 5

20 問4. 次の行列のランクを求めよ。

$$A = \begin{pmatrix} 3 & -4 & -3 \\ -2 & 2 & 1 \\ 2 & -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 2 & 4 & 2 \\ -1 & 1 & 3 & 2 \\ 1 & 2 & 3 & 1 \\ -2 & -1 & 0 & 1 \end{pmatrix}$$

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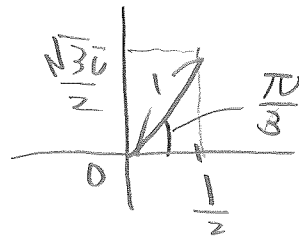
問5 次の1次方程式系を解け。

$$(1) \begin{cases} x_1 - x_2 - 2x_3 = 2 \\ 3x_1 - x_2 + 2x_3 = 8 \\ x_1 - x_2 + 2x_3 = 6 \end{cases}, \quad (2) \begin{cases} 2x_1 + 3x_2 + 2x_3 + x_4 = 1 \\ 4x_1 + 2x_2 - x_3 + x_4 = 2 \\ -2x_1 - x_2 - x_3 - 2x_4 = -1 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \end{cases}$$

数学C 解答

①

問1. (1) $\frac{1+\sqrt{3}i}{2} =$
 $= \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$



(2) $\left(\frac{1+\sqrt{3}i}{2}\right)^{-6}$
 $= \frac{1}{\left(\cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)\right)^6}$
 $= \frac{1}{\left(\cos\left(\frac{6}{3}\pi\right) + i\sin\left(\frac{6}{3}\pi\right)\right)}$
 $= 1$

問2

$$(1) A+B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}}}$$

$$(2) 2A-3C$$

$$= \begin{pmatrix} 4 & 0 \\ 2 & 6 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 3 & 6 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}}}$$

$$(3) ABC$$

$$= \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} C$$

$$= \begin{pmatrix} 2 & 4 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 6 & 8 \\ 6 & 10 \end{pmatrix}}}$$

問3

$$(1) P^{-1} = \frac{1}{-3+4} \begin{pmatrix} -1 & -2 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 2 & 3 \end{pmatrix}$$

$$(2) P^{-1}AP = \begin{pmatrix} -1 & -2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 6 & 6 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$$

$$(3) (P^{-1}AP)^m = \begin{pmatrix} 2^m & 0 \\ 0 & 3^m \end{pmatrix}$$

$$(P^{-1}AP)^m = P^{-1}AP \cdot P^{-1}AP \cdots P^{-1}AP$$

$$= P^{-1}A^mP$$

$$\therefore A^m = P \cdot (P^{-1}AP)^m \cdot P^{-1}$$

$$= \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} 2^m & 0 \\ 0 & 3^m \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \cdot 2^m & 2 \cdot 3^m \\ -2 \cdot 2^m & -1 \cdot 3^m \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 2 & 3 \end{pmatrix}$$

$$= \left(\begin{array}{cc|cc} -3 \cdot 2^m + 4 \cdot 3^m & 6 \cdot 2^m + 6 \cdot 3^m \\ 2 \cdot 2^m - 2 \cdot 3^m & -4 \cdot 2^m - 3 \cdot 3^m \end{array} \right)$$

問4. $A = 3 \times 3 = 3$

$$A = \begin{pmatrix} 3 & -4 & -3 \\ -2 & 2 & 1 \\ 2 & -1 & 0 \end{pmatrix} \xrightarrow{\substack{\textcircled{1} + \textcircled{2} \\ \textcircled{3} + \textcircled{2}}} \begin{pmatrix} 1 & -2 & -2 \\ -2 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow{\substack{\textcircled{1} + \textcircled{3} \\ \textcircled{2} - \textcircled{3}}} \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow{\textcircled{2} + 2 \times \textcircled{1}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\xrightarrow{\textcircled{3} - \textcircled{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

 $B = 3 \times 4 = 2$

$$B = \begin{pmatrix} 0 & 2 & 4 & 2 \\ -1 & 1 & 3 & 2 \\ 1 & 2 & 3 & 1 \\ -2 & -1 & 0 & 1 \end{pmatrix} \xrightarrow{\substack{\textcircled{1} \times \frac{1}{2} \\ \textcircled{2} + \textcircled{3} \\ \textcircled{4} + 2 \times \textcircled{3}}} \begin{pmatrix} 0 & 1 & 2 & 1 \\ 0 & 3 & 6 & 3 \\ 1 & 2 & 3 & 1 \\ 0 & 3 & 6 & 3 \end{pmatrix}$$

$$\xrightarrow{\substack{\textcircled{2} - 3 \times \textcircled{1} \\ \textcircled{4} - 3 \times \textcircled{1} \\ \textcircled{3} - 2 \times \textcircled{1}}} \begin{pmatrix} 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\textcircled{3} - 2 \times \textcircled{1}} \begin{pmatrix} 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\quad} \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

問5.

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(1)

$$\left(\begin{array}{ccc|c} 1 & -1 & -2 & 2 \\ 3 & -1 & 2 & 8 \\ 1 & -1 & 2 & 6 \end{array} \right) \begin{array}{l} \longrightarrow \\ \textcircled{2} - 3 \times \textcircled{1} \\ \textcircled{3} - \textcircled{1} \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -2 & 2 \\ 0 & 2 & 8 & 2 \\ 0 & 0 & 4 & 4 \end{array} \right) \begin{array}{l} \longrightarrow \\ \textcircled{2} \times \frac{1}{2} \\ \textcircled{3} \times \frac{1}{4} \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -2 & 2 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} \longrightarrow \\ \textcircled{1} + 2 \times \textcircled{3} \\ \textcircled{2} - 4 \times \textcircled{3} \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array} \right) \begin{array}{l} \longrightarrow \\ \textcircled{1} + \textcircled{2} \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$$



(2)

$$\left(\begin{array}{cccc|c} 2 & 3 & 2 & 1 & 1 \\ 4 & 2 & -1 & 1 & 2 \\ -2 & -1 & -1 & -2 & -1 \\ 2 & 1 & 2 & 3 & 1 \end{array} \right)$$



- ② $-2 \times$ ①
- ③ $+ \text{①}$
- ④ $- \text{①}$

$$\left(\begin{array}{cccc|c} 2 & 3 & 2 & 1 & 1 \\ 0 & -4 & -5 & -1 & 0 \\ 0 & 2 & 1 & -1 & 0 \\ 0 & -2 & 0 & 2 & 0 \end{array} \right)$$



- ② $-2 \times$ ④
- ③ $+ \text{④}$

$$\left(\begin{array}{cccc|c} 2 & 3 & 2 & 1 & 1 \\ 0 & 0 & -5 & -5 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & -2 & 0 & 2 & 0 \end{array} \right)$$



- ② $+ 5 \times$ ③
- ④ $\times (-\frac{1}{2})$

$$\left(\begin{array}{cccc|c} 2 & 3 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{array} \right)$$



- ① $-2 \times$ ③ $-3 \times$ ④

$$\left(\begin{array}{cccc|c} 2 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{array} \right)$$



- ① $\times \frac{1}{2}$
- 打ち消

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

(t:任意)

$$\begin{cases} x_1 + x_4 = \frac{1}{2} \\ x_2 - x_4 = 0 \\ x_3 + x_4 = 0 \end{cases}$$

$x_4 = t$ とおく

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$