

数学C・中間試験問題（午前クラス、平成29年12月20日）

20 問1. (1) $\frac{1-\sqrt{3}i}{2}$ を極形式で表せ。

(2) $\left(\frac{1-\sqrt{3}i}{2}\right)^{12}$ をもとめよ。

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15 問2. 行列 $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}$ に対して次を求めよ。

(1) $A+B$ 5

(2) $2A-3C$ 5

(3) ABC 5

15 問3. 行列 $A = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$, $P = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$ に対して次を求めよ。

(1) P^{-1} 5

(2) $P^{-1}AP$ 5

(3) A^n ($n=1,2,3,\dots$) 5

20 問4. 次の行列のランクを求めよ。

$$A = \begin{pmatrix} -3 & 2 & 2 \\ -2 & 2 & 1 \\ 2 & -1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & -7 & 6 & 1 \\ 1 & 0 & 5 & 2 \\ -1 & 5 & 5 & 3 \\ 0 & 1 & 2 & 1 \end{pmatrix}$$

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問5 次の1次方程式系を解け。

(1)
$$\begin{cases} x_1 - 2x_2 - 2x_3 = 4 \\ 3x_1 - 5x_2 - 7x_3 = 11 \\ 2x_1 - x_2 - 7x_3 = 5 \end{cases}$$

(2)
$$\begin{cases} x_1 + x_3 + 2x_4 = 6 \\ -2x_1 + x_2 + 4x_3 + x_4 = 3 \\ 4x_1 - 3x_2 - 4x_3 + x_4 = -3 \\ -x_1 + x_2 + 2x_3 + x_4 = 4 \end{cases}$$

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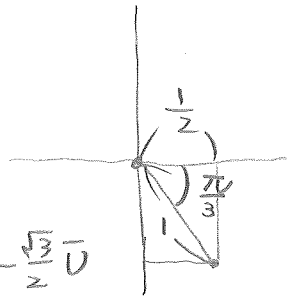
(解答) 数C・中間

①

問1.

$$(1) \frac{1-\sqrt{3}i}{2}$$

$$= \cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right) = \frac{1-\sqrt{3}i}{2}$$



$$(2) \left(\frac{1-\sqrt{3}i}{2}\right)^{12}$$

$$= \cos\left(-\frac{12}{3}\pi\right) + i\sin\left(-\frac{12}{3}\pi\right)$$

$$= 1$$

問2

$$(1) A+B = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}}}$$

$$(2) 2A-3C$$

$$= \begin{pmatrix} 4 & 0 \\ 2 & 6 \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 3 & 6 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}}}$$

$$(3) ABC$$

$$= \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} C$$

$$= \begin{pmatrix} 2 & 4 \\ 1 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 6 & 8 \\ 6 & 10 \end{pmatrix}}}$$

問 3.

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$$(1) \quad P^{-1} = \frac{1}{3} \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$$

$$(2) \quad P^{-1}AP = \frac{1}{3} \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \\ = \frac{1}{3} \begin{pmatrix} 2 & -2 \\ 5 & 10 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix}}}$$

$$(3) \quad (P^{-1}AP)^m = \begin{pmatrix} 2^m & 0 \\ 0 & 5^m \end{pmatrix}$$

$$(P^{-1}AP)^m = P^{-1}AP P^{-1}AP \dots P^{-1}AP \\ = P^{-1}A^m P$$

$$\therefore A^m = P (P^{-1}AP)^m P^{-1}$$

$$= \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 2^m & 0 \\ 0 & 5^m \end{pmatrix} \frac{1}{3} \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix} \\ = \frac{1}{3} \left(\begin{array}{cc|cc} 2 \cdot 2^m + 5^m & & -2 \cdot 2^m + 2 \cdot 5^m & \\ -2^m + 5^m & & 2^m + 2 \cdot 5^m & \end{array} \right)$$

問4

A: rank = 3

$$\begin{pmatrix} -3 & 2 & 2 \\ -2 & 2 & 1 \\ 2 & -1 & -1 \end{pmatrix} \xrightarrow{\substack{\textcircled{1} + \textcircled{3} \\ \textcircled{2} + \textcircled{3}}} \begin{pmatrix} -1 & 1 & 1 \\ 0 & 1 & 0 \\ 2 & -1 & -1 \end{pmatrix}$$

$$\xrightarrow{\substack{\textcircled{1} - \textcircled{2} \\ \textcircled{3} + \textcircled{2}}} \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & -1 \end{pmatrix} \xrightarrow{\textcircled{3} + \textcircled{1}} \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\substack{\textcircled{1} + \textcircled{3} \\ \text{交換}}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

B: rank = 2

$$\begin{pmatrix} 4 & -7 & 6 & 1 \\ 1 & 0 & 5 & 2 \\ -1 & 5 & 5 & 3 \\ 0 & 1 & 2 & 1 \end{pmatrix} \xrightarrow{\substack{\textcircled{1} + 4 \times \textcircled{3} \\ \textcircled{2} + \textcircled{3}}} \begin{pmatrix} 0 & 13 & 26 & 13 \\ 0 & 5 & 10 & 5 \\ -1 & 5 & 5 & 3 \\ 0 & 1 & 2 & 1 \end{pmatrix}$$

$$\xrightarrow{\substack{\textcircled{1} - 13 \times \textcircled{4} \\ \textcircled{2} - 5 \times \textcircled{4} \\ \textcircled{3} - 5 \times \textcircled{4}}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & -5 & -2 \\ 0 & 1 & 2 & 1 \end{pmatrix} \xrightarrow{} \begin{pmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

問5 (1)

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$$\left(\begin{array}{ccc|c} 1 & -2 & -2 & 4 \\ 3 & -5 & -7 & 11 \\ 2 & -1 & -7 & 5 \end{array} \right) \begin{array}{l} \xrightarrow{\quad} \\ \textcircled{2} - 3 \times \textcircled{1} \\ \textcircled{3} - 2 \times \textcircled{1} \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & -2 & 4 \\ 0 & -1 & -1 & -1 \\ 0 & 3 & -3 & -3 \end{array} \right) \begin{array}{l} \xrightarrow{\quad} \\ \textcircled{3} + 3 \times \textcircled{2} \\ \textcircled{1} + \textcircled{2} \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -4 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\begin{cases} x_1 - 4x_3 = 2 \\ x_2 - x_3 = -1 \end{cases} \quad x_3 = t \text{ とおく.}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$$

($t = \text{任意}$)

問5 (2)

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$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 6 \\ -2 & 1 & 4 & 1 & 3 \\ 4 & -3 & -4 & 1 & -3 \\ -1 & 1 & 2 & 1 & 4 \end{array} \right)$$



$$\textcircled{2} + 2 \times \textcircled{1}$$

$$\textcircled{3} - 4 \times \textcircled{1}$$

$$\textcircled{4} + \textcircled{1}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 6 \\ 0 & 1 & 6 & 5 & 15 \\ 0 & -3 & -8 & -7 & -27 \\ 0 & 1 & 3 & 3 & 10 \end{array} \right)$$



$$\textcircled{2} - \textcircled{4}$$

$$\textcircled{3} + 3 \times \textcircled{4}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 6 \\ 0 & 0 & 3 & 2 & 5 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 3 & 10 \end{array} \right)$$



$$\textcircled{1} - \textcircled{3}$$

$$\textcircled{2} - 3 \times \textcircled{3}$$

$$\textcircled{4} - 3 \times \textcircled{3}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & -4 & -4 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -3 & 1 \end{array} \right)$$



$$\textcircled{2} \times \left(-\frac{1}{4}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & -3 & 1 \end{array} \right)$$



$$\textcircled{3} - 2 \times \textcircled{2}$$

$$\textcircled{4} + 3 \times \textcircled{2}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 4 \end{array} \right)$$



問5 (2)

$$\left(\begin{array}{ccc|c} 1 & & & 3 \\ & 1 & & 4 \\ & 0 & 1 & \\ & & & \\ & & & \\ & & & \end{array} \right)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \\ 1 \end{pmatrix}$$
