

問 次の行列の階数 (rank) を求めなさい。

$$A = \begin{pmatrix} 1 & 1 & 4 & 2 \\ 2 & -1 & -1 & 1 \\ -1 & 1 & 2 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 2 & 2 & -1 \\ 1 & 2 & 0 & 2 & 1 \\ -1 & 1 & 1 & 0 & 0 \\ 1 & 5 & 1 & 4 & 2 \end{pmatrix}$$

$$\text{rank } A = \boxed{(1)}$$

$$\text{rank } B = \boxed{(2)}$$

$\boxed{(1)} = 2 \quad \boxed{(2)} = 3$

$A = \begin{pmatrix} 1 & 1 & 4 & 2 \\ 2 & -1 & -1 & 1 \\ -1 & 1 & 2 & 0 \end{pmatrix} \xrightarrow[\textcircled{3} + \textcircled{1}]{\textcircled{2} - 2 \times \textcircled{1}} \begin{pmatrix} 1 & 1 & 4 & 2 \\ 0 & -3 & -9 & -3 \\ 0 & 2 & 6 & 2 \end{pmatrix}$

$\xrightarrow[\textcircled{3} \times \frac{1}{2}]{\textcircled{2} \times (-\frac{1}{3})} \begin{pmatrix} 1 & 1 & 4 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 1 & 3 & 1 \end{pmatrix} \xrightarrow[\textcircled{3} - \textcircled{2}]{\textcircled{1} - \textcircled{2}} \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

rank A = 2

$B = \begin{pmatrix} 1 & 0 & 2 & 2 & -1 \\ 1 & 2 & 0 & 2 & 1 \\ -1 & 1 & 1 & 0 & 0 \\ 1 & 5 & 1 & 4 & 2 \end{pmatrix} \xrightarrow[\textcircled{4} - \textcircled{1}]{\textcircled{2} - \textcircled{1}, \textcircled{3} + \textcircled{1}} \begin{pmatrix} 1 & 0 & 2 & 2 & -1 \\ 0 & 2 & -2 & 0 & 2 \\ 0 & 1 & 3 & 2 & -1 \\ 0 & 5 & -1 & 2 & 3 \end{pmatrix}$

$\xrightarrow[\textcircled{4} - 5 \times \textcircled{3}]{\textcircled{2} - 2 \times \textcircled{3}} \begin{pmatrix} 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & -8 & -4 & 4 \\ 0 & 1 & 3 & 2 & -1 \\ 0 & 0 & -16 & -8 & 8 \end{pmatrix} \xrightarrow[\textcircled{4} \times (-\frac{1}{8})]{\textcircled{2} \times (-\frac{1}{4})} \begin{pmatrix} 1 & 0 & 2 & 2 & -1 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 3 & 2 & -1 \\ 0 & 0 & 2 & 1 & -1 \end{pmatrix}$

$\xrightarrow[\textcircled{3} - \frac{3}{2} \times \textcircled{2}]{\textcircled{1} - \textcircled{2}, \textcircled{4} - \textcircled{2}} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2} \times \textcircled{2}} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

$\xrightarrow{\textcircled{2} \leftrightarrow \textcircled{3}} \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

rank B = 3