

# 数学Ⅱ 2018 (9回目)

①

問.  $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 4 & 1 \\ 2 & -4 & 0 \end{pmatrix}$  を対角化する.

$$P^{-1}AP = \begin{pmatrix} \boxed{(1)} & & 0 \\ & \boxed{(1)} & \\ 0 & & \boxed{(2)} \end{pmatrix}.$$

$(P^{-1}AP)^n = P^{-1}A^nP$  であるから,

$$A^n = P (P^{-1}AP)^n P^{-1}$$

$$= \begin{pmatrix} \boxed{(3)} & 2^{n+1}-2 & 2^n-1 \\ -2^n+1 & 3 \cdot 2^{n+1}-2 & 2^n-1 \\ 2^{n+1}-2 & -2^{n+2} + \boxed{(4)} & -2^n+2 \end{pmatrix}$$

$$\boxed{(1)} = 2$$

$$\boxed{(2)} = 1$$

$$\boxed{(3)} = 1$$

$$\boxed{(4)} = 4$$

2

(略解)

$$P = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & -2 & -2 \end{pmatrix}$$

$$P^{-1} = \begin{pmatrix} 0 & 2 & 1 \\ -1 & 3 & 1 \\ 1 & -2 & -1 \end{pmatrix}$$

$$P^{-1}AP = \begin{pmatrix} 2 & & \\ & 2 & \\ & & 1 \end{pmatrix}$$

$$A^n = P \cdot (P^{-1}AP)^n \cdot P^{-1}$$

$$= \begin{pmatrix} 1 & 2^{n+1}-2 & 2^n-1 \\ -2^n+1 & 3 \cdot 2^n-2 & 2^n-1 \\ 2^{n+1}-2 & -2^{n+2}+4 & -2^n+2 \end{pmatrix}$$