数学Ⅱ 2018 (レポート4回目)

問1. $\mathbb{R}^4$ の部分空間 W_1, W_2 を次の式で定める。

$$W_1 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \in \mathbb{R}^4 \;\middle|\; \begin{array}{l} x_1 + x_2 + 2x_3 + 3x_4 = 0 \\ 3x_2 + 3x_3 - 2x_4 = 0 \end{array} \right\}$$

$$W_2 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \in \mathbb{R}^4 \;\middle|\; \begin{array}{l} x_1 + 2x_2 + 3x_3 + 2x_4 = 0 \\ x_1 + 3x_2 + 4x_3 + 2x_4 = 0 \end{array} \right\}$$

$$\dim W_1 = \boxed{(1)}$$

$$\dim W_2 = \boxed{(2)}$$

$$\dim W_1 \cap W_2 = \boxed{(3)}$$

問2.

$$A = \begin{pmatrix} 1 & 2 & 1 & -3 & 0 \\ -1 & -1 & 0 & 1 & 1 \\ 2 & 3 & 1 & -4 & -1 \end{pmatrix}$$

$$\dim \operatorname{Ker} A = \boxed{(4)}$$

$\boxed{(1)} = 2$

$\boxed{(2)} = 2$

$\boxed{(3)} = 1$

$\boxed{(4)} = 3$

問1.

W_1：拡大係数行列を変形

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 3 & 3 & -2 & 0 \end{array}\right) \xrightarrow[①-\frac{1}{3}②,\ \frac{1}{3}②]{}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & \frac{7}{3} & 0 \\ 0 & 1 & 1 & -\frac{2}{3} & 0 \end{array}\right)$$

$$\begin{cases} x_1 = -t - 7s \\ x_2 = -t + 2s \end{cases} \qquad x_3 = t,\ x_4 = 3s$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = t\begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} -7 \\ 2 \\ 0 \\ 3 \end{pmatrix}$$

$$W_1 = \left\{ t\begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} -7 \\ 2 \\ 0 \\ 3 \end{pmatrix} \;\middle|\; s, t \in \mathbb{R} \right\}$$

$$\underline{\dim W_1 = 2}$$

W_2：拡大係数行列を変形

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 1 & 3 & 4 & 2 & 0 \end{array}\right) \xrightarrow[②-①]{}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{array}\right) \xrightarrow[①-2\times②]{}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x_1 = -t - 2s \\ x_2 = -t \end{cases} \quad x_3 = t,\ x_4 = s$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = t \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} -2 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$W_2 = \left\{ t \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} -2 \\ 0 \\ 0 \\ 1 \end{pmatrix} \,\middle|\, s, t \in \mathbb{R} \right\}$$

$$\underline{\dim W_2 = 2}$$

$W_1 \cap W_2$

$$= \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \in \mathbb{R}^4 \,\middle|\, \begin{array}{r} x_1 + x_2 + 2x_3 + 3x_4 = 0 \\ 3x_2 + 3x_3 - 2x_4 = 0 \\ x_1 + 2x_2 + 3x_3 + 2x_4 = 0 \\ x_1 + 3x_2 + 4x_3 + 2x_4 = 0 \end{array} \right\}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 3 & 3 & -2 & 0 \\ 1 & 2 & 3 & 2 & 0 \\ 1 & 3 & 4 & 2 & 0 \end{array}\right) \xrightarrow[\substack{③-① \\ ④-①}]{}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 0 \\ 0 & 3 & 3 & -2 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 2 & 2 & -1 & 0 \end{array}\right) \xrightarrow[\substack{②-③\cdot③ \\ ④-2\times③}]{①-③}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \xrightarrow[\substack{①-4② \\ ③+②}]{}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$\begin{cases} x_1 = -t \\ x_2 = -t \\ x_3 = t \\ x_4 = 0 \end{cases}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = t \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

$$W_1 \cap W_2 = \left\{ t \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix} \middle| t \in \mathbb{R} \right\}$$

$$\underline{\dim W_1 \cap W_2 = 1}$$

問2、

$$\left(\begin{array}{ccccc|c} 1 & 2 & 1 & -3 & 0 & 0 \\ -1 & -1 & 0 & 1 & 1 & 0 \\ 2 & 3 & 1 & -4 & -1 & 0 \end{array}\right) \xrightarrow[\substack{③-2\times①}]{} $$

$② + ①$

$③ - 2\times①$

$$\left(\begin{array}{ccccc|c} 1 & 2 & 1 & -3 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & -1 & -1 & 2 & -1 & 0 \end{array}\right) \xrightarrow{}$$

$① - 2\times②$

$③ + ②$

$$\left(\begin{array}{ccccc|c} 1 & 0 & -1 & 1 & -2 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{cases} x_1 - x_3 + x_4 - 2x_5 = 0 \\ x_2 + x_3 - 2x_4 + x_5 = 0 \end{cases}$$

$x_3 = s,\ x_4 = t,\ x_5 = u$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = s\begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + u\begin{pmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathrm{Ker}\,A = \left\{ s\begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} + u\begin{pmatrix} 2 \\ -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \,\middle|\, s, t, u \in \mathbb{R} \right\}$$

$$\dim \mathrm{Ker}\,A = 3$$