数学Ⅱ 2018 (レポート2回目)

問

$$V=\mathbb{R}^3$$

$$u_1=\begin{pmatrix}1\\0\\1\end{pmatrix},\ u_2=\begin{pmatrix}2\\1\\0\end{pmatrix},\ u_3=\begin{pmatrix}1\\1\\-2\end{pmatrix}\in V$$

$$v_1=\begin{pmatrix}3\\-1\\4\end{pmatrix},\ v_2=\begin{pmatrix}4\\1\\8\end{pmatrix},\ v_3=\begin{pmatrix}3\\-2\\6\end{pmatrix}\in V$$

(1) $u_1, u_2, u_3, v_1, v_2, v_3$ は1次独立か。

1次独立ならば、1を、

1次従属ならば、2を、

どちらでもないならば、3を $\boxed{(1)}$ に入力しなさい。

(2)

$$v_1=c_1u_1+c_2u_2+c_3u_3$$

$$c_1\cdot c_2\cdot c_3=\boxed{(2)}$$

↑
積

(3)

$$v_2=d_1u_1+d_2u_2+d_3u_3$$

$$d_1\cdot d_2\cdot d_3=\boxed{(3)}$$

(4)

$$v_3=e_1u_1+e_2u_2+e_3u_3$$

$$e_1\cdot e_2\cdot e_3=\boxed{(4)}$$

2

$\boxed{(1)} = 2$

$\boxed{(2)} = -12$

$\boxed{(3)} = 168$

$\boxed{(4)} = -24.$

$$\begin{cases} v_1 = c_1 u_1 + c_2 u_2 + c_3 u_3 \\ v_2 = d_1 u_1 + d_2 u_2 + d_3 u_3 \\ v_3 = e_1 u_1 + e_2 u_2 + e_3 u_3 \end{cases}$$

行列で書くと、

$$(v_1, v_2, v_3) = (u_1, u_2, u_3) \begin{pmatrix} c_1 & d_1 & e_1 \\ c_2 & d_2 & e_2 \\ c_3 & d_3 & e_3 \end{pmatrix}$$

つまり、

$$\begin{pmatrix} 3 & 4 & 3 \\ -1 & 1 & -2 \\ 4 & 8 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} c_1 & d_1 & e_1 \\ c_2 & d_2 & e_2 \\ c_3 & d_3 & e_3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 4 & 3 \\ -1 & 1 & -2 \\ 4 & 8 & 6 \end{pmatrix}$$

Aの逆行列は、

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -2 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\text{①}-\text{③}]{}$$

$$\left(\begin{array}{ccc|ccc} 0 & 2 & 3 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -2 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\text{①}-2\times\text{②}]{}$$

$$\left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & -2 & -1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -2 & 0 & 0 & 1 \end{array}\right) \xrightarrow[\substack{\text{②}-\text{①} \\ \text{③}+2\times\text{①}}]{}$$

$$\left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & -2 & -1 \\ 0 & 1 & 0 & -1 & 3 & 1 \\ 1 & 0 & 0 & 2 & -4 & -1 \end{array}\right) \xrightarrow[\text{行いれかえ}]{}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -4 & -1 \\ 0 & 1 & 0 & -1 & 3 & 1 \\ 0 & 0 & 1 & 1 & -2 & -1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 2 & -4 & -1 \\ -1 & 3 & 1 \\ 1 & -2 & -1 \end{pmatrix}.$$

$$B = A \begin{pmatrix} c_1 & d_1 & e_1 \\ c_2 & d_2 & e_2 \\ c_3 & d_3 & e_3 \end{pmatrix}$$

$$\therefore \begin{pmatrix} c_1 & d_1 & e_1 \\ c_2 & d_2 & e_2 \\ c_3 & d_3 & e_3 \end{pmatrix} = A^{-1} \cdot B$$

$$= \begin{pmatrix} 2 & -4 & -1 \\ -1 & 3 & 1 \\ 1 & -2 & -1 \end{pmatrix} \begin{pmatrix} 3 & 4 & 3 \\ -1 & 1 & -2 \\ 4 & 8 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 6 & -4 & 8 \\ -2 & 7 & -3 \\ 1 & -6 & 1 \end{pmatrix}$$

よって
$$\begin{cases} c_1 c_2 c_3 = -12 = \boxed{(2)} \\ d_1 d_2 d_3 = 168 = \boxed{(3)} \\ e_1 e_2 e_3 = -24 = \boxed{(4)} \end{cases}$$