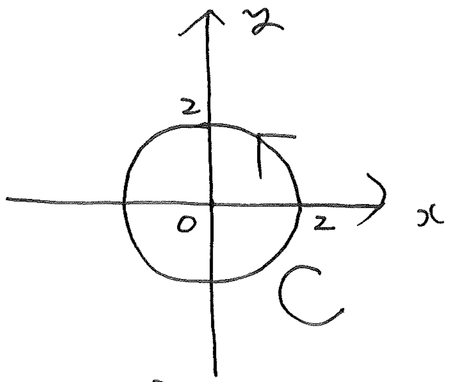
数III 2018 レポート（6回目）

問 Cauchyの積分表示

$$\frac{2\pi i}{n!} f^{(n)}(z) = \int_C \frac{f(\zeta)}{(\zeta - z)^{n+1}} d\zeta$$

を活用して次の積分を求めよ。

ただし C は、原点を中心とする
半径2の反時計回りの円とする。



(1) $I_1 = \int_C \frac{\cos\zeta}{\zeta} d\zeta = 2\pi i \times \boxed{(1)}$

(2) $I_2 = \int_C \frac{e^\zeta}{\zeta^2} d\zeta = 2\pi i \times \boxed{(2)}$

(3) $I_3 = \int_C \frac{\zeta e^\zeta}{(\zeta-1)^4} d\zeta = \frac{\boxed{(3)}}{\boxed{(4)}} \times \pi e i$

既約分数

②

$\boxed{(1)} = 1$

$\boxed{(2)} = 1$

$\boxed{(3)} = 4$

$\boxed{(4)} = 3$

③

(1) $f(\xi) = \cos\xi$

$z = 0$, $n = 0$ とおけば、

$$\int_C \frac{f(\xi)}{(\xi - z)^{n+1}} d\xi = \int_C \frac{\cos\xi}{\xi} d\xi = I_1$$

よって

$$I_1 = 2\pi i f(0) = 2\pi i.$$

(2) $f(\xi) = e^{\xi}$

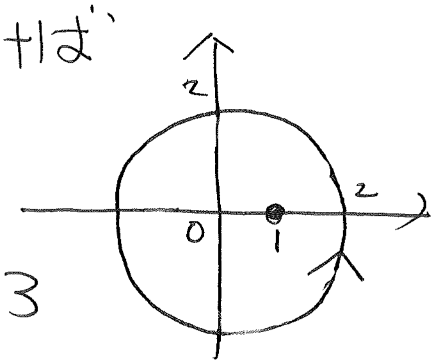
$z = 0$, $n = 1$ とおけば、

$$\int_C \frac{f(\xi)}{(\xi - z)^{n+1}} d\xi = \int_C \frac{e^{\xi}}{\xi^2} d\xi = I_2$$

よって $I_2 = 2\pi i f'(0) = 2\pi i e^0 = 2\pi i$

(3) $f(\xi) = \xi e^{\xi}$

$z = 1$, $n = 3$ とおけば



$$\int_C \frac{f(\xi)}{(\xi - z)^{n+1}} d\xi = \int \frac{\xi e^{\xi}}{(\xi - 1)^4} d\xi = I_3$$

よって

$$I_3 = \frac{2\pi i}{3!} \times f^{(3)}(1)$$

$$f(\xi) = \xi e^{\xi}$$

$$f'(\xi) = (\xi+1)e^{\xi}$$

$$f''(\xi) = e^{\xi} + (\xi+1)e^{\xi} = (\xi+2)e^{\xi}$$

$$f^{(3)}(\xi) = e^{\xi} + (\xi+2)e^{\xi} = (\xi+3)e^{\xi}$$

$$I_3 = \frac{2\pi i}{6} \times (1+3) e^{1} = \frac{4}{3}\pi i \times e$$