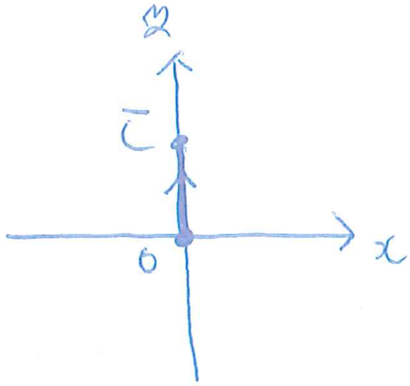
数III 2018 （4回目）

問1.

$$f(z) = \frac{1}{(1-z)^2}$$

C



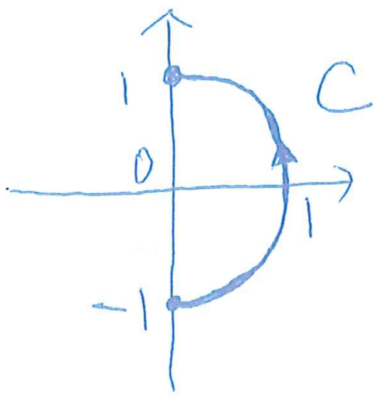

$$\int_C f(z)\,dz = \frac{1}{\boxed{(1)}} + \frac{i}{\boxed{(2)}}$$

原点0からiへ向かう線分

問2.

$$f(z) = \mathrm{Log}\, z$$

$$C : z(\theta) = e^{i\theta} \quad \left(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right)$$



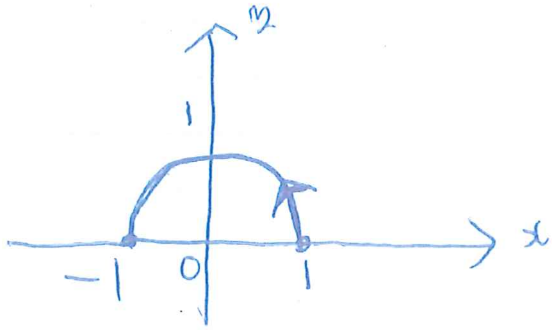

$$\int_C f(z)\,dz = \boxed{(3)} \times i$$

2

問3.

$$f(z) = |z|$$

$$C : z(\theta) = e^{i\theta} \quad (0 \leq \theta \leq \pi)$$




$$\int_C f(z)\,dz = \boxed{(4)}$$

3

$\boxed{(1)} = -2$

$\boxed{(2)} = 2$

$\boxed{(3)} = -2$

$\boxed{(4)} = -2$

問1.

$$\int_C f(z)\,dz$$

$F(z)=\dfrac{1}{1-z}$ とおく.

$$\frac{d}{dz}F(z)=\frac{1}{(1-z)^2}=f(z).$$

$\boxed{(1)}=-2$

$\boxed{(2)}=2$

よって

$$\int_C f(z)\,dz=\left[\frac{1}{1-z}\right]_0^i=\frac{1}{1-i}-1=-\frac{1}{2}+\frac{i}{2}$$

問2.

主値

$$\int_C f(z)\,dz=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\mathrm{Log}\,e^{i\theta}\,\frac{d}{d\theta}(e^{i\theta})\,d\theta$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} i\theta\cdot ie^{i\theta}\,d\theta$$

$$=-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\theta e^{i\theta}$$

$u=\theta,\ u'=1$
$v'=-e^{i\theta},\ v=ie^{i\theta}$

$$=\left[i\theta e^{i\theta}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ie^{i\theta}\,d\theta$$

$$=\left(i\frac{\pi}{2}i-i\left(-\frac{\pi}{2}\right)(-i)\right)-\left[e^{i\theta}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$=\left(-\frac{\pi}{2}+\frac{\pi}{2}\right)-\left(i-(-i)\right)=-2i$$

$\boxed{(3)}=-2$

問3



$$\int_C f(z)dz$$

$$= \int_0^{\pi} |e^{i\theta}| ie^{i\theta} d\theta$$

$$= \int_0^{\pi} ie^{i\theta} d\theta$$

$$= \left[ e^{i\theta} \right]_0^{\pi}$$

$$= -1-1 = -2$$