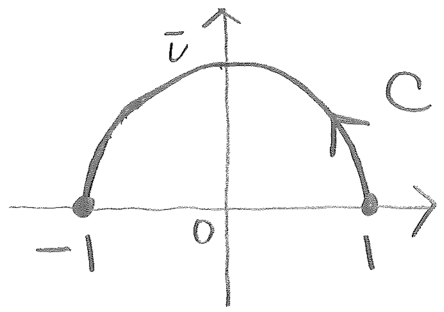
数学III レポート（3回目）

問1.

$f(z) = |z|$, $g(z) = 1/z$.

曲線 C は単位円の上半部とする.

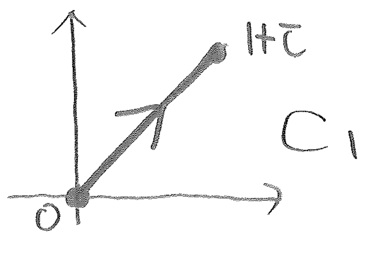


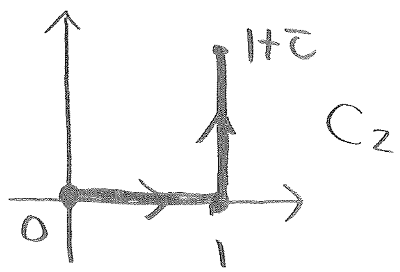
$$\int_C f(z)\,dz = \boxed{(1)}$$

$$\int_C g(z)\,dz = \boxed{(2)} \times \pi i$$

問2. $f(z) = z^2 + iz + 1$

曲線 C_1, C_2 を次の図で定める.





$$\int_{C_1} f(z)\,dz = -\frac{2}{3} + \frac{\boxed{(3)}}{3} i$$

$$\int_{C_2} f(z)\,dz = -\frac{2}{3} + \frac{\boxed{(4)}}{3} i$$

$\boxed{(1)} = -2$

$\boxed{(2)} = 1$

$\boxed{(3)} = 5$

$\boxed{(4)} = 5$

問1.

曲線Cのパラメータ表示は、

$$z(\theta)=e^{i\theta} \qquad (0\leq\theta\leq\pi).$$

$$\int_C f(z)dz=\int_0^{\pi}|e^{i\theta}|\frac{d}{d\theta}(e^{i\theta})d\theta$$

$$=i\int_0^{\pi}e^{i\theta}d\theta=i\left[\frac{1}{i}e^{i\theta}\right]_0^{\pi}=-1-1=\underline{-2}$$

$$\int_C g(z)dz=\int_0^{\pi}\frac{1}{e^{i\theta}}\frac{d}{d\theta}(e^{i\theta})d\theta$$

$$=i\int_0^{\pi}d\theta=\underline{\pi i}$$

問2.

C_1のパラメータ表示は、

$$z=z_1(t)=(1+i)t \quad (0\leq t\leq 1).$$

C_2の 〃

$$z=z_2(t)=\begin{cases} t & (0\leq t\leq 1) \\ 1+it & (0\leq t\leq 1) \end{cases}.$$

よって

$$\int_{C_1}f(z)dz=\int_0^1\{((1+i)t)^2+i(1+i)t+1\}(1+i)dt$$

$$=\int_0^1(2i\cdot t^2+(i-1)t+1)(1+i)dt$$

$$=\left[\frac{2i}{3}t^3+\frac{1}{2}(i-1)t^2+t\right]_0^1\times(1+i)$$

$$=\underline{-\frac{2}{3}+\frac{5}{3}i}.$$

$$\int_{C_2} f(z)\,dz = \int_0^1 (t^2 + it + 1)\,dt$$

$$+ \int_0^1 \underbrace{\{(1+it)^2 + i(1+it) + 1\}}_{= -t^2 + (2i-1)t + i + 2} i\,dt$$

$$= \left[\frac{t^3}{3} + \frac{i}{2}t^2 + t\right]_0^1$$

$$+ i\left[-\frac{1}{3}t^3 + \frac{1}{2}(2i-1)t^2 + (i+2)t\right]_0^1$$

$$= -\frac{2}{3} + \frac{5}{3}i$$