数学Ⅲ 2018　2回目レポート

問　次の複素数 を $a+bi$ $(a, b \in \mathbb{R})$ の形に表せ.

$$\mathrm{Log}(-1+\sqrt{3}i) = \log 2 + \frac{\boxed{(1)}}{\boxed{(2)}}\pi i$$

既約分数

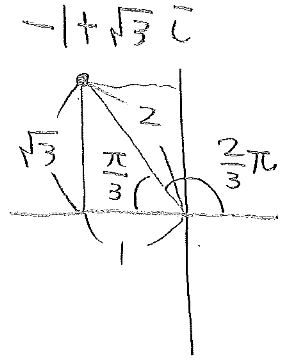
$$(-i)^{i} = e^{\frac{\pi}{\boxed{(3)}} + 2\pi m} \quad (m \in \mathbb{Z})$$

$$(1+i)^{\frac{2}{3}} = 2^{\frac{1}{3}}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right),$$

$$2^{\frac{1}{3}}\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right),$$

$$2^{\frac{1}{3}}\left(\cos\frac{\boxed{(4)}}{6}\pi + i\sin\frac{\boxed{(4)}}{6}\pi\right).$$

（解答）



$$\mathrm{Log}(-1+\sqrt{3}i)$$
$$=\log 2+\frac{2}{3}\pi i$$

$\boxed{(1)}=2$, $\boxed{(2)}=3$,

$$(-i)^{i}$$
$$=e^{i\log(-i)}$$
$$=e^{i(-\frac{\pi}{2}i+2\pi i n)}$$
$$=e^{\frac{\pi}{2}+2\pi n}$$

$$\log(-i)=\log 1-\frac{\pi}{2}i+2\pi i n$$
$$=-\frac{\pi}{2}i+2\pi i n$$

$(n\in\mathbb{Z})$ $\boxed{(3)}=2$

$$(1+i)^{\frac{2}{3}}$$

$$\log(1+i)=\log\sqrt{2}+\frac{\pi}{4}i+2\pi i n$$

$$=e^{\frac{2}{3}\log(1+i)}$$
$$=e^{\frac{2}{3}(\log\sqrt{2}+\frac{\pi}{4}i+2\pi i n)}$$
$$=e^{\frac{1}{3}\log 2+\frac{\pi}{6}i+\frac{4}{3}\pi i n}$$

$\boxed{(4)}=17$.

$$=2^{\frac{1}{3}}\left(\cos\left(\frac{\pi}{6}+\frac{4}{3}\pi n\right)+i\sin\left(\frac{\pi}{6}+\frac{4}{3}\pi n\right)\right)$$

$$= 2^{\frac{1}{3}}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right),$$

$$2^{\frac{1}{3}}\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right),$$

$$2^{\frac{1}{3}}\left(\cos\frac{17}{6}\pi + i\sin\frac{17}{6}\pi\right).$$