

数学Ⅲ 2018 2回目レポート

①

問 次の複素数を $a+b\bar{i}$ ($a, b \in \mathbb{R}$)
の形に表せ.

$$\text{Log}(-1+\sqrt{3}\bar{i}) = \log 2 + \frac{\boxed{(1)}}{\boxed{(2)}} \pi \bar{i}$$

$$(-\bar{i})^{\bar{i}} = e^{\frac{\pi}{\boxed{(3)}} + 2\pi m} \quad (m \in \mathbb{Z})$$

↑
既約分数

$$(1+\bar{i})^{\frac{2}{3}} = 2^{\frac{1}{3}} \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right),$$

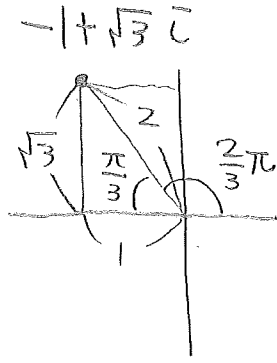
$$2^{\frac{1}{3}} \left(\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi \right),$$

$$2^{\frac{1}{3}} \left(\cos \frac{\boxed{(4)}}{6} \pi + i \sin \frac{\boxed{(4)}}{6} \pi \right).$$

(2)

(解答)

$$\begin{aligned} & \text{Log} (-1 + \sqrt{3}i) \\ &= \log 2 + \frac{2}{3}\pi i \end{aligned}$$



$$\boxed{(1)} = 2, \quad \boxed{(2)} = 3,$$

$$\begin{aligned} & (-i)^{\bar{i}} \\ &= e^{\bar{i} \log(-i)} \\ &= e^{-i(-\frac{\pi}{2}i + 2\pi i n)} \\ &= e^{\frac{\pi}{2} + 2\pi n} \end{aligned}$$

$$\begin{aligned} \log(-i) &= \log |-\frac{\pi}{2}i + 2\pi i n| \\ &= -\frac{\pi}{2}i + 2\pi i n \end{aligned}$$

$$(n \in \mathbb{Z}) \quad \boxed{(3)} = 2$$

$$\begin{aligned} & (1+i)^{\frac{2}{3}} \\ &= e^{\frac{2}{3} \log(1+i)} \\ &= e^{\frac{2}{3} (\log \sqrt{2} + \frac{\pi}{4}i + 2\pi i n)} \end{aligned}$$

$$\log(1+i) = \log \sqrt{2} + \frac{\pi}{4}i + 2\pi i n$$

$$= e^{\frac{1}{3} \log 2 + \frac{\pi}{6}i + \frac{4}{3}\pi i n}$$

$$\boxed{(4)} = 17$$

$$= 2^{\frac{1}{3}} \left(\cos\left(\frac{\pi}{6} + \frac{4}{3}\pi n\right) + i \sin\left(\frac{\pi}{6} + \frac{4}{3}\pi n\right) \right)$$

3

$$= 2^{\frac{1}{3}} \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right),$$

$$2^{\frac{1}{3}} \left(\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi \right),$$

$$2^{\frac{1}{3}} \left(\cos \frac{11}{6}\pi + i \sin \frac{11}{6}\pi \right).$$