数学Ⅱレポート（6回目）

問、次の重積分を計算しなさい。

(i) $f(x,y) = 1$

$D = \{(x,y) \mid 0 \le y \le 1, 0 \le x \le 4-y\}$

$$\iint_D f(x,y)\,dxdy = \frac{\boxed{(1)}}{2}.$$

(ii) $f(x,y) = xy$

$D = \{(x,y) \mid -1 \le y \le 1, 0 \le x \le 2-y\}$

$$\iint_D f(x,y)\,dxdy = -\frac{4}{\boxed{(2)}}$$

(iii) $f(x,y) = \cos(x+y+z)$

$D = \{(x,y,z) \mid 0 \le x \le \pi, 0 \le y \le 2\pi, 0 \le z \le 1\}$

$$\iiint_D f(x,y)\,dxdydz = \boxed{(3)}$$

(iv) $f(x,y) = x\sin y$

$D = \{(x,y) \mid 0 \le y \le \pi, 0 \le x \le \cos y\}$

$$\iint_D f(x,y)\,dxdy = \frac{1}{\boxed{(4)}}$$

$\boxed{(1)} = 7$

$\boxed{(2)} = 3$

$\boxed{(3)} = 0$

$\boxed{(4)} = 3.$

(i)

$$\iint_D f(x,y)\,dx\,dy$$

$$= \int_0^1 \left(\int_0^{4-y} 1\,dx \right) dy$$

$$= \int_0^1 \left[x \right]_{x=0}^{x=4-y} dy$$

$$= \int_0^1 (4-y)\,dy$$

$$= \left[4y - \frac{1}{2}y^2 \right]_{y=0}^{y=1}$$

$$= \frac{7}{2}$$

(ii) $\iint_D f(x,y)\,dx\,dy$

$$= \int_{-1}^1 \left(\int_0^{2-y} xy\,dx \right) dy$$

$$= \int_{-1}^1 \left[\frac{1}{2} y x^2 \right]_{x=0}^{x=2-y} dy$$

$$= \int_{-1}^1 \frac{1}{2}(2-y)^2 y\,dy$$

$$= \int_{-1}^1 \left(\frac{1}{2}y^3 - 2y^2 + 2y \right) dy$$

$$= \left[2y^4 - \frac{2}{3}y^3 + y^2 \right]_{y=-1}^{y=1}$$

$$= -\frac{4}{3}$$

(iii) $$\iiint_D f(x,y,z)\,dx\,dy\,dz$$

$$= \int_0^1 \left(\int_0^{2\pi} \left(\int_0^{\pi} \cos(x+y+z)\,dx \right) dy \right) dz$$

$$= \int_0^1 \left(\int_0^{2\pi} \Big[+\sin(x+y+z) \Big]_{x=0}^{x=\pi} dy \right) dz$$

$$= \int_0^1 \left(\int_0^{2\pi} (-2)\sin(y+z)\,dy \right) dz$$

$$= \int_0^1 \Big[2\cos(y+z) \Big]_{y=0}^{y=2\pi} dz$$

$$= \int_0^1 0\,dz = 0$$

(iv) $$\iint_D f(x,y)\,dx\,dy$$

$$= \int_0^{\pi} \left(\int_0^{\cos y} x \sin y\,dx \right) dy$$

$$= \int_0^{\pi} \Big[\frac{1}{2} x^2 \sin y \Big]_{x=0}^{x=\cos y} dy$$

$$= \int_0^{\pi} \frac{1}{2} \sin y \cdot \cos^2 y\,dy$$

$$= \Big[-\frac{1}{6} \cos^3 y \Big]_{y=0}^{y=\pi}$$

$$= -\frac{1}{6}(-1-1) = \frac{1}{3}$$