数学I 2018　（レポート3回目）

問1、

次の関数の2次偏導関数を求めよ。

$$f(x,y)=\sqrt{1-x-y}$$

$$\frac{\partial^2 f}{\partial x \partial y}(x,y)=(-)\frac{1}{\boxed{(1)}\sqrt{(1-x-y)^3}}$$

問2、

$$f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$$

$$\frac{\partial f}{\partial x}(0,y)=\boxed{(2)}\cdot y$$

$$\frac{\partial f}{\partial y}(x,0)=\boxed{(3)}\cdot x$$

$$\frac{\partial^2 f}{\partial y \partial x}(0,0)=\cancel{\frac{\partial^2 f}{\partial x \partial y}}(0,0)=-1$$

$$\frac{\partial^2 f}{\partial x \partial y}(0,0)=\frac{\partial^2 f}{\partial y \partial x}(0,0)=\boxed{(4)}$$

2

$\boxed{(1)} = 4$

$\boxed{(2)} = -1$

$\boxed{(3)} = 1$

$\boxed{(4)} = 1$

問1. $\dfrac{\partial f}{\partial y}(x,y)=\dfrac{1}{2}\dfrac{-1}{\sqrt{1-x-y}}$

$$\frac{\partial^2 f}{\partial x\partial y}(x,y)=\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\frac{-1}{\sqrt{(1-x-y)^3}}$$

$$=-\frac{1}{4\sqrt{(1-x-y)^3}}$$

問2.

$(x,y)\neq(0,0)$ のとき

$$\begin{cases}\dfrac{\partial f}{\partial x}(x,y)=\dfrac{y(x^4+4x^2y^2-y^4)}{(x^2+y^2)^2}\\[2ex] \dfrac{\partial f}{\partial y}(x,y)=\dfrac{x(x^4-4x^2y^2-y^4)}{(x^2+y^2)^2}\end{cases}$$

$$\begin{cases}\dfrac{\partial f}{\partial x}(0,0)=\displaystyle\lim_{\Delta x\to 0}\dfrac{f(\Delta x,0)-f(0,0)}{\Delta x}=0\\[2ex] \dfrac{\partial f}{\partial y}(0,0)=\displaystyle\lim_{\Delta y\to 0}\dfrac{f(0,\Delta y)-f(0,0)}{\Delta y}=0\end{cases}$$

$x,y\neq 0$ のとき

$$\begin{cases}\dfrac{\partial f}{\partial x}(0,y)=\dfrac{-y^5}{y^4}=-y\\[2ex] \dfrac{\partial f}{\partial y}(x,0)=\dfrac{x^5}{x^4}=x\end{cases}$$

まとめて、$x, y \in \mathbb{R}$ に対し、

$$\begin{cases} \dfrac{\partial f}{\partial x}(0, y) = -y \\ \dfrac{\partial f}{\partial y}(x, 0) = x \end{cases}$$

よって

$$\frac{\partial^2 f}{\partial y \partial x}(0,0) = -1$$

$$\frac{\partial^2 f}{\partial x \partial y}(0,0) = 1$$