

確率統計 2017 レポート (5回目)

①

問1.

1月1日生まれの人が510人の中に1人もいない確率は

$$\left(\frac{364}{365}\right)^{\boxed{(1)}} \quad \text{である.}$$

2項分布のポアソン近似により、

$$\left(\frac{364}{365}\right)^{\boxed{(1)}} \doteq \boxed{(2)} \% \quad \begin{array}{l} \uparrow \\ \text{整数} \end{array}$$

次のページのポアソン分布表を用いてもよい。

問2.

サイコロを200回ふる。1の目が20回以下しかでない確率を中心極限定理により求めよ。

$$P(X \leq 20) = \int_{-\infty}^{20} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$$

$$\text{ただし, } \mu = \frac{100}{3}, \quad \sigma = \frac{5}{3} \sqrt{\boxed{(3)}}.$$

$$P(X \leq 20) = 0. \quad \boxed{(4)} \% \quad \begin{array}{l} \uparrow \\ \text{2桁の整数} \end{array}$$

付 表

ポアソン分布表 $\Pr\{X=x\} = e^{-\lambda} \frac{\lambda^x}{x!}$

x	λ										x
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
0	.904837	.818731	.740818	.670320	.606531	.548812	.496585	.449329	.406570	.367879	0
1	.090484	.163746	.222245	.268128	.303265	.329287	.347610	.359463	.365913	.367879	1
2	.004524	.016375	.033337	.053626	.075816	.098786	.121663	.143785	.164661	.183940	2
3	.000151	.001092	.003334	.007150	.012636	.019757	.028388	.038343	.049398	.061313	3
4	.000004	.000055	.000250	.000715	.001580	.002964	.004968	.007669	.011115	.015328	4
5	—	.000002	.000015	.000057	.000158	.000356	.000696	.001227	.002001	.003066	5
6	—	—	.000001	.000004	.000013	.000036	.000081	.000164	.000300	.000511	6
7	—	—	—	—	.000001	.000003	.000008	.000019	.000039	.000073	7
8	—	—	—	—	—	—	.000001	.000002	.000004	.000009	8
9	—	—	—	—	—	—	—	—	—	.000001	9

x	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	x
0	.332871	.301194	.272532	.246597	.223130	.201897	.182684	.165299	.149569	.135335	0
1	.366158	.361433	.354291	.345236	.334695	.323034	.310562	.297538	.284180	.270671	1
2	.201387	.216860	.230289	.241665	.251021	.258428	.263978	.267784	.269971	.270671	2
3	.073842	.086744	.099792	.112777	.125510	.137828	.149587	.160671	.170982	.180447	3
4	.020307	.026023	.032432	.039472	.047067	.055131	.063575	.072302	.081216	.090224	4
5	.004467	.006246	.008432	.011052	.014120	.017642	.021615	.026029	.030862	.036089	5
6	.000819	.001249	.001827	.002579	.003530	.004705	.006124	.007809	.009773	.012030	6
7	.000129	.000214	.000339	.000516	.000756	.001075	.001487	.002008	.002653	.003437	7
8	.000018	.000032	.000055	.000090	.000142	.000215	.000316	.000452	.000630	.000859	8
9	.000002	.000004	.000008	.000014	.000024	.000038	.000060	.000090	.000133	.000191	9
10	—	.000001	.000001	.000002	.000004	.000006	.000010	.000016	.000025	.000038	10
11	—	—	—	—	—	.000001	.000002	.000003	.000004	.000007	11
12	—	—	—	—	—	—	—	—	.000001	.000001	12

x	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0	x
0	.122456	.110803	.100259	.090718	.082085	.074274	.067206	.060810	.055023	.049787	0
1	.257159	.243767	.230595	.217723	.205212	.193111	.181455	.170268	.159567	.149361	1
2	.270016	.268144	.265185	.261268	.256516	.251045	.244964	.238375	.231373	.224042	2
3	.189012	.196639	.203308	.209014	.213763	.217572	.220468	.222484	.223660	.224042	3
4	.099231	.108151	.116902	.125409	.133602	.141422	.148816	.155739	.162154	.168031	4
5	.041677	.047587	.053775	.060196	.066801	.073539	.080360	.087214	.094049	.100819	5
6	.014587	.017448	.020614	.024078	.027834	.031867	.036162	.040700	.045457	.050409	6
7	.004376	.005484	.006773	.008255	.009941	.011836	.013948	.016280	.018832	.021604	7
8	.001149	.001508	.001947	.002477	.003106	.003847	.004708	.005698	.006827	.008102	8
9	.000268	.000369	.000498	.000660	.000863	.001111	.001412	.001773	.002200	.002701	9
10	.000056	.000081	.000114	.000158	.000216	.000289	.000381	.000496	.000638	.000810	10
11	.000011	.000016	.000024	.000035	.000049	.000068	.000094	.000126	.000168	.000221	11
12	.000002	.000003	.000005	.000007	.000010	.000015	.000021	.000029	.000041	.000055	12
13	—	.000001	.000001	.000001	.000002	.000003	.000004	.000006	.000009	.000013	13
14	—	—	—	—	—	.000001	.000001	.000001	.000002	.000003	14
15	—	—	—	—	—	—	—	—	—	.000001	15

$$\boxed{(1)} = 510$$

$$\boxed{(2)} = 25$$

$$\boxed{(3)} = 10$$

$$\boxed{(4)} = 57$$

問2.

まともなサイコロを200回投げ、
1の目が20回以下しかでない確率を
中心極限定理により概算しなさい。

1の目がでる回数を x とすると、

x は二項分布 $Bm(200, \frac{1}{6})$ に従う。

確率密度は $f(x) = {}_{200}C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{200-x}$.

もよめたい確率は、 $P = \sum_{x=0}^{20} f(x)$.

$n=200$ は十分大きいと考え、中心極限定理
により、正規分布で概算される。

$$P \doteq \int_{-\infty}^{20} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx$$

$$\text{ただし } \mu = np = 200 \times \frac{1}{6} = \frac{100}{3}$$

$$\sigma^2 = np(1-p) = \frac{1000}{36} = \left(\frac{5}{3}\sqrt{10}\right)^2$$

変数変換 $z = \frac{x-\mu}{\sigma}$ により、

$$P \doteq \int_{-\infty}^{\frac{20 - \frac{100}{3}}{\frac{5}{3}\sqrt{10}}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right) dz$$

$$\doteq \int_{2.53}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right) dz$$

積分数値表により、

$$P \doteq 0.0057$$

約 0.57%