

数学 C 2017 (L11-ト4回目)

問1. $X = \begin{pmatrix} x & y \\ z & -x \end{pmatrix}$ とする.

$$X^{100} = (x^2 + yz)^p E_2$$

$$p = \boxed{(1)}$$

問2. $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -1 & -2 \end{pmatrix}$ とする.

$$AB = \begin{pmatrix} 0 & \boxed{(2)} \\ \boxed{(2)} & 0 \end{pmatrix}$$

(2)

$$\boxed{(1)} = 50$$

$$\boxed{(2)} = 0$$

問2.

$$\begin{aligned} AB &= \begin{pmatrix} 12 & 24 \\ 24 & -1-2 \end{pmatrix} \\ &= \left(\begin{array}{cc|cc} 2-2 & 4-4 & & \\ 4-4 & 8-8 & & \end{array} \right) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \end{aligned}$$

問1.

$$\begin{aligned} X^2 &= \begin{pmatrix} x & y \\ z & -x \end{pmatrix} \begin{pmatrix} x & y \\ z & -x \end{pmatrix} = \begin{pmatrix} x^2+yz & 0 \\ 0 & x^2+yz \end{pmatrix} \\ &= (x^2+yz) E_2 \end{aligned}$$

よって

$$\begin{aligned} X^{100} &= (X^2)^{50} = \{(x^2+yz) E_2\}^{50} \\ &= (x^2+yz)^{50} \cdot E_2 \end{aligned}$$