微積分解法 2017 レポート（9回目）

問 次の積分を求めよ.

$$\int \frac{x^3}{(x-1)^2(x^2+1)}\,dx = -\frac{1}{a(x-1)} + \log|x-1|$$

$$+\frac{1}{b}\,\mathrm{Arctan}\,x + C$$

$$a = \boxed{(1)}$$

$$b = \boxed{(2)}$$

$\boxed{(1)} = 2$

$\boxed{(2)} = 2$

まず、被積分関数の部分分数展開を求める.

$$\frac{x^3}{(x-1)^2(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1} .$$

通分して.

$$x^3 = A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x-1)^2$$
$$= x^3(A+C) + x^2(-A+B-2C+D)$$
$$+ x(A+C-2D) + (-A+B+D)$$

$\therefore$ $A+C=1,\ -A+B-2C+D=0$

$A+C-2D=0,\ -A+B+D=0$

$\therefore$ $\underline{A=1,\ B=\frac{1}{2},\ C=0,\ D=\frac{1}{2}}$

$$\int \frac{x^3}{(x-1)^2(x^2+1)}dx = \int\left(\frac{1}{x-1} + \frac{\frac{1}{2}}{(x-1)^2} + \frac{\frac{1}{2}}{x^2+1}\right)dx$$
$$= \log|x-1| - \frac{1}{2}\frac{1}{x-1} + \frac{1}{2}\operatorname{Arctan} x + C$$

$\therefore$ $\boxed{(1)} = a = 2$

$\boxed{(2)} = b = 2$