微積分解法2017 レポート 7回目

問1. 次の極限を求めよ.

$$\lim_{n\to\infty} n\left(\frac{1}{n^2+1}+\frac{1}{n^2+2^2}+\cdots+\frac{1}{n^2+n^2}\right)$$

$$= \frac{\pi}{\boxed{(1)}}$$

問2.

$$\int x^3 e^x dx = e^x\left(x^3-3x^2+\boxed{(2)}\,x-\boxed{(2)}\right)$$
$$+C.$$

$\boxed{(1)} = 4$

$\boxed{(2)} = 6.$

問1.

$$\lim_{n\to\infty} n\left(\frac{1}{n^2+1}+\frac{1}{n^2+2^2}+\cdots+\frac{1}{n^2+n^2}\right)$$

$$=\lim_{n\to\infty}\frac{1}{n}\left(\frac{1}{1+\frac{1}{n^2}}+\frac{1}{1+\frac{2^2}{n^2}}+\cdots+\frac{1}{1+\frac{n^2}{n^2}}\right)$$

$$=\int_0^1 \frac{1}{1+x^2}\,dx$$

$$=\Big[\mathrm{Arctan}\,x\Big]_0^1=\frac{\pi}{4}$$

問2.

$u=x^3 \quad u'=3x^2$
$v'=e^x \quad v=e^x$

$$\int x^3e^x\,dx$$

$$=x^3e^x-3\int x^2e^x\,dx$$

$u=x^2 \quad u'=2x$
$v'=e^x \quad v=e^x$

$$=x^3e^x-3\left(x^2e^x-2\int xe^x\,dx\right)$$

$u=x \quad u'=1$
$v'=e^x \quad v=e^x$

$$=x^3e^x-3x^2e^x+6\int xe^x\,dx$$

$$=x^3e^x-3x^2e^x+6\left(xe^x-\int e^x\,dx\right)$$

$$=x^3e^x-3x^2e^x+6xe^x-6e^x+C.$$