

問. 次の微分方程式を解け.

(1)

$$\frac{dy}{dx} = \frac{(1+x^2)y^3}{(y^2-1)x^3}$$

$$x^a + y^a = 2 \log \left(C \left| \frac{x}{y} \right| \right)$$

$$a = \boxed{(1)}$$

(2)

$$(1+y^2) + (1+x^2) \frac{dy}{dx} = 0$$

$$\text{Arctan } x + b \text{ Arctan } y = C$$

$$b = \boxed{(2)}$$

$$\boxed{(1)} = -2$$

$$\boxed{(2)} = 1$$

(1)

$$\frac{y^2-1}{y^3} \frac{dy}{dx} = \frac{1+x^2}{x^3}$$

$$\int \left(\frac{1}{y} - \frac{1}{y^3} \right) dy = \int \left(\frac{1}{x^3} + \frac{1}{x} \right) dx$$

$$\log|y| + \frac{1}{2}y^{-2} = -\frac{1}{2}x^{-2} + \log|x| + C'$$

$$\log \left(e^{C'} \left| \frac{x}{y} \right| \right) = \frac{1}{2} (x^{-2} + y^{-2})$$

$$\therefore \underline{2 \log \left(c \left| \frac{x}{y} \right| \right) = x^{-2} + y^{-2}}$$

(2)

$$\frac{1}{1+y^2} \frac{dy}{dx} = -\frac{1}{1+x^2}$$

$$\int \frac{1}{1+y^2} dy = -\int \frac{1}{1+x^2} dx$$

$$\text{Arctan } y = -\text{Arctan } x + C$$

$$\underline{\text{Arctan } x + \text{Arctan } y = C}$$