微積分解法 2017 レポート (10回目)

問1. 次の積分を求めよ.

$$\int \mathrm{Arctan}\, x \, dx = x \,\mathrm{Arctan}\, x - \frac{1}{a} \log(1+x^2) + C$$

$$\boxed{(1)} = a.$$

問2.

$$I = \int \frac{\sin x}{1+\sin x} dx = x - \int \frac{1}{1+\sin x} dx$$

$$= x + \frac{b}{1+\tan\frac{x}{2}} + C$$

$$\boxed{(2)} = b.$$

ヒント: $t = \tan\frac{x}{2}$ とおく.

$\boxed{(1)} = 2$

$\boxed{(2)} = 2$

問1.

$$\int \mathrm{Arctan}\, x \, dx \qquad \begin{cases} f = \mathrm{Arctan}\, x \\ g' = 1 \end{cases}$$

$$= x\,\mathrm{Arctan}\, x - \int \frac{x}{1+x^2} dx \qquad \begin{cases} f' = \frac{1}{1+x^2} \\ g = x \end{cases}$$

$$= x\,\mathrm{Arctan}\, x - \frac{1}{2}\log(x^2+1) + C$$

問2.

$$I = \int \frac{\sin x}{1+\sin x} dx$$

$$= \int \frac{1+\sin x - 1}{1+\sin x} dx = \int \left(1 - \frac{1}{1+\sin x}\right) dx$$

$$= x - \int \frac{1}{1+\sin x} dx$$

$t = \tan \frac{x}{2}$ とおく.

$$\frac{dt}{dx} = \frac{1+t^2}{2}, \quad \sin x = \frac{2t}{1+t^2}$$

$$\int \frac{1}{1+\sin x} dx = \int \frac{1}{1+\frac{2t}{1+t^2}} \frac{2}{1+t^2} dt = \int \frac{2}{(1+t)^2} dt$$

$$= -\frac{2}{1+t} + C'$$

まとめて、

$$I = x + \frac{2}{1+\tan\frac{x}{2}} + C$$