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## 微積分解法2016 レポート4回目

問 次の  $f(x), g(x)$  のマクローリン展開を  
 もとめなさい。

$$\begin{cases} f(x) = \sin^2 x, \\ g(x) = \frac{1}{\sqrt{1+x}}. \end{cases}$$

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{a^{2n+1}}{(an+a)!} x^{2n+2}$$

$$\boxed{(1)} = a$$

$$g(x) = \sum_{n=0}^{\infty} \frac{(bn-1)!!}{(bn)!!} x^n$$

$$\boxed{(2)} = b$$

$$\boxed{(1)} = 2$$

$$\boxed{(2)} = 2$$

$$f(x) = \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos y = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} y^{2n} \quad \text{これを使えば、}$$

$$= -\frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} (2x)^{2n}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{2^{2n+1}}{(2n+2)!} x^{2n+2}$$

$$g(x) = \frac{1}{\sqrt{1-x}}$$

$$\left\{ \begin{aligned} g'(x) &= -\frac{1}{2} (1+x)^{-\frac{3}{2}} \times (-1) \\ g^{(2)}(x) &= \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) (1+x)^{-\frac{5}{2}} \times (-1)^2 \\ &\vdots \\ g^{(n)}(x) &= \left(-\frac{1}{2}\right) \left(-\frac{3}{2}\right) \cdots \left(-\frac{1}{2} - n + 1\right) (1+x)^{-\frac{1}{2} - n} (-1)^n \end{aligned} \right.$$

$$g^{(n)}(0) = \frac{(2n-1)!!}{2^n}$$

$$g(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \cdot \frac{1}{2^n} \cdot (2n-1)!! x^n$$
$$= \sum_{n=0}^{\infty} \frac{(2n-1)!!}{(2n)!!} x^n$$

(別解)

$$f(x) = \sin^2 x$$

$$f'(x) = 2 \sin x \cos x = \sin(2x)$$

$$\sin^{(n)}(x) = \sin(x + \frac{\pi}{2}n) \text{ であるから}$$

$n=1, 2, 3, \dots$  のとき、

$$f^{(n)}(x) = 2^{n-1} \sin(x + \frac{\pi}{2}(n-1))$$

$$f^{(n)}(0) = 2^{n-1} \sin \frac{\pi}{2}(n-1)$$

$$f^{(n)}(0) = 2^{n-1} \times \begin{cases} \sin 2\pi k = 0 & (n = 4k+1) \\ \sin(\frac{\pi}{2} + 2\pi k) = 1 & (n = 4k+2) \\ \sin(\pi + 2\pi k) = 0 & (n = 4k+3) \\ \sin(\frac{3}{2}\pi + 2\pi k) = -1 & (n = 4k+4) \end{cases}$$

$(k=0, 1, 2, \dots)$

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また、

$$f^{(10)}(0) = f(0) = \sin^2 0 = 0.$$

よって、

$$\begin{aligned} f(x) = \sin^2 x &= \sum_{k=0}^{\infty} \frac{2^{4k+1}}{(4k+2)!} x^{4k+2} \\ &\quad - \sum_{k=0}^{\infty} \frac{2^{4k+3}}{(4k+4)!} x^{4k+4} \\ &= \sum_{m=0}^{\infty} \frac{(-1)^m 2^{2m+1}}{(2m+2)!} x^{2m+2} \end{aligned}$$