変数分離微分方程式

(1) $\dfrac{dy}{dx} = 2x \cdot y^2$

(2) $\dfrac{dy}{dx} = -\dfrac{\tan y \cdot \cos^2 y}{\tan x \cdot \cos^2 x}$

(3) $\dfrac{dy}{dx} = -\dfrac{\tan x}{\tan y}$

(4) $\dfrac{dy}{dx} = \dfrac{(1+x^2)y^3}{(y^2-1)x^3}$

(5) $x^2\dfrac{dy}{dx} + y^2 = 0$

(6) $y\dfrac{dy}{dx} = x(1+y^2)$

(7) $(1+x)y + (1+y)x \cdot \dfrac{dy}{dx} = 0$

(8) $xy(1+x^2)\dfrac{dy}{dx} = 1-y^2$

(9) $(1+y^2) + (1+x^2)\dfrac{dy}{dx} = 0$

(1) ～ (3) 授業で説明 ⎫
(4) レポートと同じ、 ⎭ 省略、

(5)

$$\frac{1}{y^2}\frac{dy}{dx} = -\frac{1}{x^2}$$

$$\int \frac{1}{y^2}\,dy = -\int \frac{1}{x^2}\,dx$$

$$-\frac{1}{y} = \frac{1}{x} + C$$

$$\underline{Cxy + x + y = 0}$$

(6)

$$\frac{y}{1+y^2}\frac{dy}{dx} = x$$

$$\int \frac{y}{1+y^2}\,dy = \int x\,dx$$

$$\frac{1}{2}\log(1+y^2) = \frac{1}{2}x^2 + C$$

$$1+y^2 = e^{x^2}e^{2C}$$

e^{2C} を新たに C と書くと、

$$\underline{1+y^2 = Ce^{x^2}}$$

(7)

$$\frac{1+y}{y}\frac{dy}{dx} = -\frac{1+x}{x}$$

$$\int\left(\frac{1}{y}+1\right)dy = -\int\left(\frac{1}{x}+1\right)dx$$

$$\log y + y = -\log x - x + C$$

$$\underline{x+y+\log x+\log y = C}$$

(8)

$$\frac{y}{1-y^2}\frac{dy}{dx} = \frac{1}{x(x^2+1)}$$

さて

・ $$\frac{y}{1-y^2} = \frac{y}{(1+y)(1-y)} = \frac{1}{2}\left(\frac{1}{1-y}+\frac{1}{1+y}\right)$$

・ $$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$$1 = A(x^2+1) + (Bx+C)x$$

$x=0$ を代入. $1=A$

$x=i$ を代入 $1=B(-1)+Ci$

$\therefore B=-1, C=0$

$$\frac{1}{x(x^2+1)} = \frac{1}{x} - \frac{x}{x^2+1}$$

$$\int \frac{y}{1-y^2}\,dy = \int \frac{1}{x(x^2+1)}\,dx$$

$$(左辺) = \int \frac{1}{2}\left(\frac{1}{1-y} - \frac{1}{1+y}\right)dy$$

$$= \frac{1}{2}\left(-\log(1-y) - \log(1+y)\right) + C_1$$

$$= -\frac{1}{2}\log(1-y^2) + C_1$$

$$(右辺) = \int \left(\frac{1}{x} - \frac{x}{x^2+1}\right)dx$$

$$= \log x - \frac{1}{2}\log(x^2+1) + C_2$$

まとめて $-\frac{1}{2}\log(1-y^2) = \log x - \frac{1}{2}\log(x^2+1) + C_2 - C_1$

$$1-y^2 = \frac{1}{x^2}(1+x^2)e^{2(C_2-C_1)}$$

$C = e^{2(C_2-C_1)}$ とおくと、

$$\underline{x^2(1-y^2) = C(1+x^2)}$$

(9)

$$\frac{1}{1+y^2}\frac{dy}{dx} = -\frac{1}{1+x^2}$$

$$\int \frac{1}{1+y^2}\,dy = -\int \frac{1}{1+x^2}\,dx$$

$$\mathrm{Arctan}\, y = -\mathrm{Arctan}\, x + C$$

$$\mathrm{Arctan}\, x + \mathrm{Arctan}\, y = C$$