微積分解法レポート2016（1回目）

問1.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+5)} = \frac{\boxed{(1)}}{300}$$

問2.

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = \boxed{(2)}$$

（注）

$$k! = k(k-1)\cdots 2\cdot 1$$

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$\boxed{(1)} = 137$

$\boxed{(2)} = 1$

問1. $\frac{1}{n(n+5)} = \frac{1}{5}\left(\frac{1}{n} - \frac{1}{n+5}\right)$ なので

$$S_N = \sum_{n=1}^{N} \frac{1}{n(n+5)}$$

$$= \sum_{n=1}^{N} \frac{1}{5}\left(\frac{1}{n} - \frac{1}{n+5}\right)$$

$$= \frac{1}{5}\left(\sum_{n=1}^{N}\frac{1}{n} - \sum_{n=6}^{N+5}\frac{1}{n}\right)$$

$$= \frac{1}{5}\left(\sum_{n=1}^{5}\frac{1}{n} - \sum_{n=N+1}^{N+5}\frac{1}{n}\right)$$

よって $\lim_{N\to\infty} S_N = \frac{1}{5}\left(\sum_{n=1}^{5}\frac{1}{n}\right) = \underline{\frac{137}{300}}$

問2.

$\frac{1}{n!} - \frac{1}{(n+1)!} = \frac{n}{(n+1)!}$ なので

$$S_N = \sum_{n=1}^{N} \frac{n}{(n+1)!}$$

$$= \sum_{n=1}^{N}\left(\frac{1}{n!} - \frac{1}{(n+1)!}\right)$$

$$= \sum_{n=1}^{N}\frac{1}{n!} - \sum_{n=2}^{N+1}\frac{1}{n!}$$

$$= 1 - \frac{1}{(N+1)!}$$

よって $\lim_{N\to\infty} S_N = \underline{1}$