数学I 2016 レポート 9回目.

問1.

$$I = \iiint_D 1 \, dxdydz$$

$$D = \{(x,y,z) \mid x+y+z \leq 1, \ x,y,z \geq 0\}$$

$$I = \frac{\boxed{(1)}}{\boxed{(2)}}$$ ← 既約分数とする.

問2.

$$I = \iiint_D z^2 \sqrt{x^2+y^2+z^2} \, dxdydz$$

$$D = \{(x,y,z) \mid x^2+y^2+z^2 \leq R^2\} \quad (R \geq 0)$$

$$I = \frac{\boxed{(3)}}{\boxed{(4)}} \pi R^6$$ ← 既約分数

$\boxed{(1)} = 1$

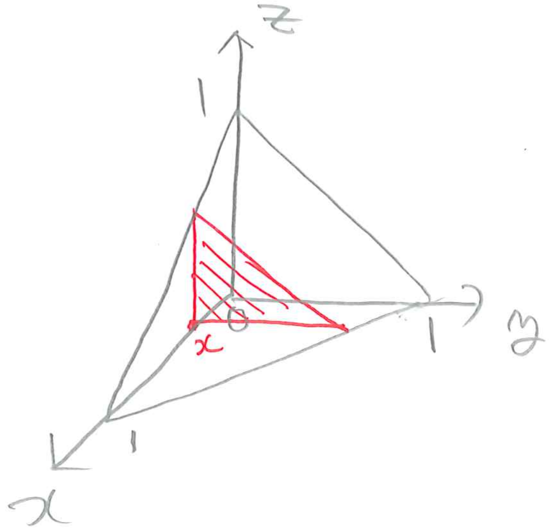
$\boxed{(2)} = 6$

$\boxed{(3)} = 2$

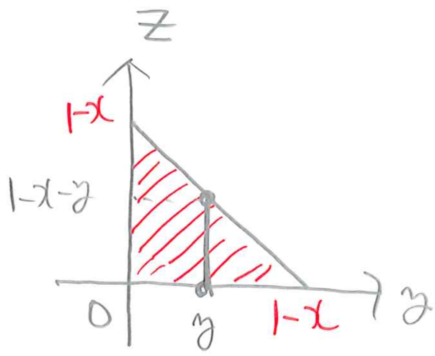
$\boxed{(4)} = 9$

問1.

$$D = \{(x, y, z) \mid x+y+z \leq 1,\ x, y, z \geq 0\}$$
$$= \{(x, y, z) \mid 0 \leq x \leq 1,\ 0 \leq y \leq 1-x,\ 0 \leq z \leq 1-x-y\}$$



断面



$$I = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} 1 \, dz \, dy \, dx$$

$$= \int_0^1 \int_0^{1-x} \Big[z \Big]_{z=0}^{z=1-x-y} dy \, dx$$

$$= \int_0^1 \int_0^{1-x} (1-x-y) \, dy \, dx$$

$$= \int_0^1 \Big[(1-x)y - \frac{1}{2}y^2 \Big]_{y=0}^{y=1-x} dx$$

$$= \int_0^1 \frac{1}{2}(1-x)^2 \, dx$$

$$= \Big[\frac{-1}{6}(1-x)^3 \Big]_{x=0}^{x=1} = \underline{\frac{1}{6}}$$

問2、

極座標に変換する、

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r\sin\varphi\cos\theta \\ r\sin\varphi\sin\theta \\ r\cos\varphi \end{pmatrix}$$

ヤコビアン $J(r,\varphi,\theta) = r^2\sin\varphi$ であるから、

$$I = \iiint_{D'} r^2\cos^2\varphi \times r \times r^2\sin\varphi \, dr\, d\varphi\, d\theta$$

$D' = \{(r,\varphi,\theta) \mid 0 \leq r \leq R, 0 \leq \varphi \leq \pi, 0 \leq \theta \leq 2\pi\}$

変数分離しており、

$$I = \int_0^R r^5 \, dr \int_0^\pi \cos^2\varphi \cdot \sin\varphi \, d\varphi \int_0^{2\pi} 1 \, d\theta$$

$$= \left[\frac{1}{6}r^6\right]_{r=0}^{r=R} \times \left[-\frac{1}{3}\cos^3\varphi\right]_{\varphi=0}^{\varphi=\pi} \left[\theta\right]_0^{2\pi}$$

$$= \frac{1}{6}R^6 \times \frac{2}{3} \times 2\pi$$

$$= \underline{\frac{2}{9}\pi R^6}$$