数学Ⅰレポート2016（7回目）

問　次の積分を計算せよ.

(1) $\displaystyle\iint_D (x+y)\,dxdy = \frac{\boxed{(1)}}{3}$

D は $x=0,\ y=0,\ x+y=2$ で囲まれた領域

(2) $\displaystyle\iint_D (x+y)\,dxdy = \frac{3}{\boxed{(2)}}$

D は $x=y,\ x=y^2$ で囲まれた領域

(3) $\displaystyle\iint_D x^2\,dxdy = \frac{\boxed{(3)}}{3}$

$D = \{(x,y) \mid |x|+|y|<1\}$

(4) $\displaystyle\iint_D \frac{x}{\cos^2 xy}\,dxdy = \frac{1}{2\pi}\log\boxed{(4)}$

$D = \{(x,y) \mid 0 \leq x \leq \frac{1}{4},\ 0 \leq y \leq \pi\}$

②

$\boxed{(1)} = 8$

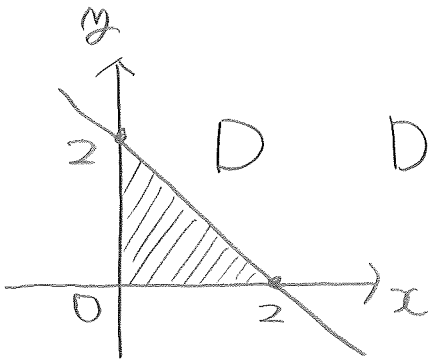
$\boxed{(2)} = 20$

$\boxed{(3)} = 1$

$\boxed{(4)} = 2$

（解答）

(1)



$D=\{(x,y)\mid 0\leq x\leq 2,\ 0\leq y\leq 2-x\}$

縦線集合、

$$\iint_D (x+y)\,dxdy$$

$$=\int_0^2\int_0^{2-x}(x+y)\,dy\,dx$$

$$=\int_0^2\left[xy+\frac{1}{2}y^2\right]_{y=0}^{y=2-x}dx$$

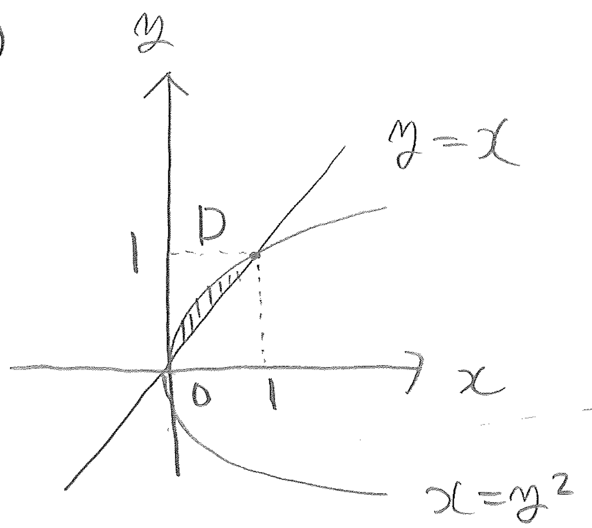
$$=\int_0^2\left\{(2-x)x+\frac{1}{2}(2-x)^2\right\}dx$$

$$=\int_0^2\left(-\frac{1}{2}x^2+2\right)dx$$

$$=\left[-\frac{1}{6}x^3+2x\right]_{x=0}^{x=2}$$

$$=-\frac{8}{6}+4=\underline{\frac{8}{3}}$$

(2)



$D = \{(x, y) \mid 0 \leq y \leq 1,\ y^2 \leq x \leq y\}$ 縦線集合.

$$\iint_D (x+y)\,dxdy$$

$$= \int_0^1 \int_{y^2}^{y} (x+y)\,dx\,dy$$

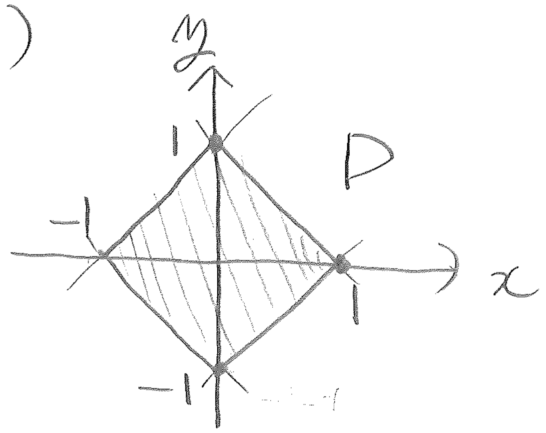
$$= \int_0^1 \left[\frac{x^2}{2} + yx\right]_{x=y^2}^{x=y} dy$$

$$= \int_0^1 \left\{\frac{3}{2}y^2 - \left(\frac{y^4}{2} + y^3\right)\right\} dy$$

$$= \left[-\frac{y^5}{10} - \frac{y^4}{4} + \frac{y^3}{2}\right]_{y=0}^{y=1}$$

$$= -\frac{1}{10} - \frac{1}{4} + \frac{1}{2} = \underline{\frac{3}{20}}$$

(3)



$$D = \{(x, y) \mid -1 \leq x \leq 0,\ -x-1 \leq y \leq x+1\}$$

$$\cup \{(x, y) \mid 0 \leq x \leq 1,\ x-1 \leq y \leq -x+1\}$$

$$\iint_D x^2\, dx\,dy = \int_{-1}^{0} \int_{-x-1}^{x+1} x^2\, dy \cdot dx$$

$$+ \int_0^1 \int_{x-1}^{-x+1} x^2\, dy\, dx$$

$$= \int_{-1}^{0} x^2 \left[y \right]_{y=-x-1}^{y=x+1} dx$$

$$+ \int_0^1 x^2 \left[y \right]_{y=x-1}^{y=-x+1} dx$$

$$= \int_{-1}^{0} x^2 (2x+2)\, dx + \int_0^1 x^2(-2x+2)\, dx$$

$$= \left[\frac{1}{2}x^4 + \frac{2}{3}x^3 \right]_{x=-1}^{x=0} + \left[-\frac{x^4}{2} + \frac{2}{3}x^3 \right]_{x=0}^{x=1}$$

$$= -\left(\frac{1}{2} - \frac{2}{3} \right) + \left(-\frac{1}{2} + \frac{2}{3} \right) = \underline{\frac{1}{3}}$$

(4)

$$\begin{cases} \displaystyle\int \frac{1}{\cos^2 t}\,dt = \tan t + C \\ \displaystyle\int \tan t\,dt = -\log|\cos t| + C \end{cases}$$

$$\iint_D \frac{x}{\cos^2 xy}\,dx\,dy$$

$$= \int_0^{\frac{1}{4}} \int_0^{\pi} \frac{x}{\cos^2 xy}\,dy\,dx$$

$$= \int_0^{\frac{1}{4}} x \left[\frac{1}{x}\tan(xy)\right]_{y=0}^{y=\pi} dx$$

$$= \int_0^{\frac{1}{4}} \tan(\pi x)\,dx$$

$$= \left[-\frac{1}{\pi}\log|\cos(\pi x)|\right]_{x=0}^{x=\frac{1}{4}}$$

$$= -\frac{1}{\pi}\log\left|\cos\left(\frac{\pi}{4}\right)\right| + \frac{1}{\pi}\log 1$$

$$= -\frac{1}{\pi}\log\frac{1}{\sqrt{2}} = \underline{\frac{1}{2\pi}\log 2}$$