数学I 2016 レポート（5回目）

問

$z=f(x,y)=xye^{-x^2-y^2}$ は、

$\pm\left(\dfrac{1}{\sqrt{\boxed{(1)}}}, \dfrac{1}{\sqrt{\boxed{(1)}}}\right)$ で極大

$\pm\left(\dfrac{1}{\sqrt{\boxed{(2)}}}, \dfrac{-1}{\sqrt{\boxed{(2)}}}\right)$ で極小

$\left(\boxed{(3)}, \boxed{(4)}\right)$ であん点

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$\boxed{(1)} = 2$

$\boxed{(2)} = 2$

$\boxed{(3)} = 0$

$\boxed{(4)} = 0$

問.

$$\begin{cases} f_x = ye^{-x^2-y^2} + xy(-2x)e^{-x^2-y^2} \\ \quad = y(1-2x^2)e^{-x^2-y^2} \\ f_y = x(1-2y^2)e^{-x^2-y^2} \end{cases}$$

$$f_{x,x} = y(-4x)e^{-x^2-y^2} + y(1-2x^2)(-2x)e^{-x^2-y^2}$$

$$= (-2xy)(3-2x^2)e^{-x^2-y^2}$$

$$f_{y,y} = (-2xy)(3-2y^2)e^{-x^2-y^2}$$

$$f_{x,y} = (1-2x^2)e^{-x^2-y^2} + y(1-2x^2)(-2y)e^{-x^2-y^2}$$

$$= (1-2x^2)(1-2y^2)e^{-x^2-y^2}$$

$f_x = 0$, $f_y = 0$ を解いて

臨界点は. $(0,0)$, $\pm\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $\pm\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

の5つ.

極値判定定理により.

$$H(0,0) = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 < 0$$ あん点.

$$H\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right) = \begin{vmatrix} -e^{-1} & 0 \\ 0 & -e^{-1} \end{vmatrix} = e^{-2} > 0$$

$$f_{xx}\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right) = -e^{-1} < 0 \quad \text{極大}$$

$$H\left(\pm\frac{1}{\sqrt{2}}, \mp\frac{1}{\sqrt{2}}\right) = \begin{vmatrix} e^{-1} & 0 \\ 0 & e^{-1} \end{vmatrix} = e^{-2} > 0$$

$$f_{xx}\left(\pm\frac{1}{\sqrt{2}}, \mp\frac{1}{\sqrt{2}}\right) = e^{-1} > 0 \quad \text{極小}$$