数学Ⅱ レポート 2016 (2回目)

問1. $f(x,y) = \log(x^2+y^2)$

$x = \cos t,\ y = t$

$$\frac{df}{dt} = \frac{\boxed{(1)}\ \sin 2t + \boxed{(2)} \times t}{\cos^2 t + t^2}$$

問2. $f(x,y)$ C^2級

$x = a+ut,\ y = b+vt$

$$\frac{df}{dt} = \boxed{(3)} \times u\frac{\partial f}{\partial x} + v\cdot\frac{\partial f}{\partial y}.$$

$$\frac{d^2f}{dt^2} = u^2\frac{\partial^2 f}{\partial x^2} + \boxed{(4)} \times uv\frac{\partial^2 f}{\partial x \partial y} + v^2\frac{\partial^2 f}{\partial y^2}.$$

$[(1)] = -1$

$[(2)] = 2$

$[(3)] = 1$

$[(4)] = 2$

問1.

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\cdot\frac{dy}{dt}$$

$$= \frac{2x}{x^2+y^2}(-\sin t) + \frac{2y}{x^2+y^2}\cdot 1$$

$$= \frac{2\cos t}{\cos^2 t + t^2}(-\sin t) + \frac{2t}{\cos^2 t + t^2}$$

$$= \frac{-\sin 2t + 2t}{\cos^2 t + t^2}$$

問2.

$\frac{dx}{dt} = u$, $\frac{dy}{dt} = v$

$$\frac{df}{dt} = \frac{\partial f}{\partial x}u + \frac{\partial f}{\partial y}v$$

$$\frac{d^2f}{dt^2} = \frac{d}{dt}\left(\frac{df}{dt}\right)$$

$$= u\frac{d}{dt}\left(\frac{\partial f}{\partial x}\right) + v\frac{d}{dt}\left(\frac{\partial f}{\partial y}\right)$$

$$= u\left(\frac{\partial^2 f}{\partial x^2}u + \frac{\partial^2 f}{\partial y \partial x}v\right)$$

$$+ v\left(\frac{\partial^2 f}{\partial x \partial y}u + \frac{\partial^2 f}{\partial y^2}\cdot v\right)$$

$$= u^2\frac{\partial^2 f}{\partial x^2} + 2uv\frac{\partial^2 f}{\partial x \partial y} + v^2\frac{\partial^2 f}{\partial y^2}$$

C^2級なので $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ となることを用いた.